Estimation of Investment in Residential and Nonresidential Structures v2.0

October 2015

In the REMI model, the investment expenditures depends on the gap between the optimal capital stocks and the actual capital stocks. The general expressions of both residential and nonresidential investment can be written as

$$I_t = \alpha(K_t^* - K_t) \tag{1}$$

where

$I_t = investment\ in\ time\ period\ t;$

$K_t^* = optimal\ capital\ stocks\ in\ time\ period\ t;$

$K_t = actual\ capital\ stocks\ in\ time\ period\ t;$

$\alpha = the\ speed\ of\ adjustment.$

The speed of adjustment measures the proportion of gap that is eliminated by investment each year and is the coefficient to be estimated. In each time period, the actual capital stocks equal the capital stocks at the end of the previous time period depreciated during the current time period, so the investment equation can be expressed as

$$I_t = \alpha[K_t^* - (1 - d_t)K_{t-1}] \tag{2}$$

where

$d_t = depreciation\ rate\ of\ time\ period\ t;$

$K_{t-1} = capital\ stocks\ of\ the\ previous\ time\ period\ t-1.$

The actual capital stocks equal the depreciated capital stocks of the last time period plus investment, such that

$$K_t = (1 - d_t)K_{t-1} + I_t \tag{3}$$

Similarly, K_{t-1} is calculated as

$$K_{t-1} = (1 - d_{t-1})K_{t-2} + I_{t-1} \tag{4}$$

Using equation (3) and (4), we replace the actual capital stocks with the depreciated capital stocks and investment of previous time periods, so that actual capital stocks can be written as

$$K_t = K_0 \prod_{i=1}^{t}(1 - d_i) + \sum_{i=1}^{t-1} I_i \cdot \prod_{j=i+1}^{t}\left(1 - d_j\right) \tag{5}$$

where

$K_0 = the\ inital\ capital\ stocks.$

Substituting equation (5) into equation (2) produces

$$I_t = \alpha \left\{ K_t^* - K_0 \prod_{i=1}^{t}(1 - d_i) + \sum_{i=1}^{t-1} I_i \cdot \left[\prod_{j=i+1}^{t}\left(1 - d_j\right) \right] \right\} \tag{6}$$

The optimal regional capital stocks for residential and nonresidential structures are calculated as shares of optimal national capital stocks. The optimal residential capital stock depends on the regional share of real disposable income and the regional capital preference factor, such that

$$K_{t,r}^{R*} = \beta_r^R \cdot \left(\frac{RYD_{t,r}}{RYD_{t,u}} \right) K_{t,u}^{*R} \tag{7}$$

where

$RYD_t = real\ disposable\ income$;

$\beta = regional\ preference\ for\ residential\ capital$;

R denotes residential capital; r denotes regional; and u denotes capital.

The optimal nonresidential capital stocks depend on the regional share of employment, labor costs and capital costs, such that

$$K_{t,r}^{N*} = \beta_r^N \cdot \left[\frac{\left(\frac{ARW_{t,r}}{ARW_{t,u}} \right) \cdot \left(\frac{AE_{t,r}}{AE_{t,u}} \right)}{\left(\frac{ARC_{t,r}}{ARC_{t,u}} \right)} \right] K_{t,u}^{*N} \tag{8}$$

where

$ARW = labor\ costs$;

$AE = employment$;

$ARC = capital\ costs$;

N denotes nonresidential capital.

Substituting equation (7) and (8) into (6), we could transform the investment equation for residential and nonresidential investment into

$$I_{t,r}^{R} = \alpha^{R}\left\{\beta^{R}\cdot\left(\frac{RYD_{t,r}}{RYD_{t,u}}\right)K_{t,u}^{*R} - K_{0}^{R}\prod_{i=1}^{t}\left(1-d_{i}^{R}\right) + \sum_{i=1}^{t-1} I_{i}^{R}\cdot\left[\prod_{j=i+1}^{t}\left(1-d_{j}^{R}\right)\right]\right\} \tag{9}$$

$$I_{t,r}^{N} = \alpha^{N}\left\{\beta^{N}\cdot\left[\frac{\left(\frac{ARW_{t,r}}{ARW_{t,u}}\right)\cdot\left(\frac{AE_{t,r}}{AE_{t,u}}\right)}{\left(\frac{ARC_{t,r}}{ARC_{t,u}}\right)}\right]K_{t,u}^{*N} - K_{0}^{N}\prod_{i=1}^{t}\left(1-d_{i}^{N}\right) + \sum_{i=1}^{t-1} I_{i}^{N}\cdot\left[\prod_{j=i+1}^{t}\left(1-d_{j}^{N}\right)\right]\right\} \tag{10}$$

where α, β and K_0 are unknown parameters to be estimated.

To solve the equations, we specify equation (1) for the nation and the optimal national capital stocks can be expressed as

$$K_{t}^{*u} = \frac{I_{t}^{u}}{\alpha} + K_{t}^{u} \tag{11}$$

Using equation (9) and (11), we could solve for the investment in residential structures. For simplicity, we let

$$A_{t}^{R} = \prod_{i=1}^{t}\left(1-d_{i}^{R}\right)$$

$$B_{t}^{R} = \sum_{i=1}^{t-1} I_{i}^{R}\cdot\left[\prod_{j=i+1}^{t}\left(1-d_{j}^{R}\right)\right]$$

$$C_{t}^{R} = \beta^{R}\left(\frac{RYD_{t,r}}{RYD_{t,u}}\right)\left[\frac{I_{t}^{u}}{\alpha^{R}} + \left(1-d_{i}^{R}\right)K_{t-1,u}^{R}\right]$$

so the investment equation for residential structures can be written as

$$I_{t,r}^R = K_0^R \cdot (-\alpha^R \cdot A_t^R) - \alpha^R B_t^R + \beta^R \cdot \alpha^R C_t^R \quad (12)$$

Similarly, we use equation (10) and (11) to solve for investment in nonresidential structures. Assuming

$$A_t^N = \prod_{i=1}^{t} (1 - d_i^N)$$

$$B_t^R = \sum_{i=1}^{t-1} I_i^R \cdot \left[\prod_{j=i+1}^{t} (1 - d_j^R) \right]$$

$$C_t^N = \beta^N \left[\frac{\left(\frac{ARW_{t,r}}{ARW_{t,u}}\right) \cdot \left(\frac{AE_{t,r}}{AE_{t,u}}\right)}{\left(\frac{ARC_{t,r}}{ARC_{t,u}}\right)} \right] \left[\frac{I_t^u}{\alpha^N} + (1 - d_i^N) K_{t-1,u}^N \right]$$

we rewrite the investment equation for nonresidential investment as

$$I_{t,r}^N = K_0^N \cdot (-\alpha^N \cdot A_t^N) - \alpha^N B_t^N + \beta^N \cdot \alpha^N C_t^N \quad (13)$$

Therefore, the general final investment equation for both residential and nonresidential investment is

$$I_t = K_0 \cdot (-\alpha \cdot A_t) - \alpha B_t + \beta \cdot \alpha C_t \quad (14)$$

Data

We use panel data of 50 states and Washing D.C. from 1999 to 2013 to estimate the investment equations for residential and nonresidential structures separately. The national capital stocks and real depreciation rate for residential and nonresidential fixed assets are from the Bureau of Economic Analysis (BEA). The real disposable income in the residential investment equation is from the REMI PI+ V1.7. We use the private nonfarm employment, relative composite labor costs, and the relative capital costs data from PI+ V1.7 for the employment share, relative labor costs, and relative capital costs in the nonresidential investment equation.

Real investment data at the national level are from the BEA private fixed investment in residential and nonresidential structures. The state-level investment data need to be constructed. For the investment in residential structures, we utilize the building permits data from Census new privately owned housing units authorized valuation to estimate the regional share of the total national investment. Census provides data of private nonresidential construction put in place by state, which is used to estimate the regional nonresidential investment expenditures.

Estimation and Result

To estimate the equation, we loop over a range of values for α. In each loop, we treat α as known and plug the value of α into the equation. Therefore, the target equation is transformed into an equation with linear parameters. We apply fixed effects models to estimate the target equation and record the least sum of squared residuals for each loop. The value of α that minimizes the sum of squared residuals is the final estimated speed of adjustment.

The estimated speed of adjustment for residential investment is 0.217. Thus, 21.7 percent of gap between optimal and actual stocks of residential capital are eliminated each year. The estimated speed of adjustment for nonresidential investment is 0.078, indicating that only 7.8 percent of gap between optimal and actual stocks of nonresidential capital are eliminated each year. The following table presents a comparison of estimated speeds of adjustment from different versions of models. Our new estimates are slightly higher for both of the two sets of regressions.

Comparison of Estimated Speeds of Adjustment

	New Estimates	**Current Model Coefficients**	**2001 Estimates**	**1993 Estimates**
Residential Investment	0.217	0.128	0.097	0.127
Non-residential Investment	0.078	0.064	0.070	0.061