REMI PI+ v2.0

REMI TranSight v4.0

REMI Tax-PI v2.0

Metro-PI v2.0

Model Equations

Table of Contents

I. Introduction

Since "all politics are local," the effects of policies on sub-national areas have always been of great interest in the policy-making process. If anything, the concern about regional economies is becoming greater. The reasons for this heightened concern have to do with a combination of economic realities, changing political structures, and the influence of economic research that has emerged over the last decade.

First, after decades of steadily expanding economic prosperity, evidence began to suggest that lagging economies may not inevitably catch up to more advanced areas. Coastal China has continued to develop more rapidly than the interior; much of the income growth in the U.S. in the past decade has been focused in leading metropolitan areas of the Northeast, Texas, and California; and regional disparities persist in almost every European country.

Second, national economies have become more open, through both globalization and regional blocks such as NAFTA and the EU. This changing political organization forces local economic regions to compete with each other, without the national protection of industries. Thus, regions within a country may have an economy that is much stronger or weaker than the national economy as a whole. For example, the states of eastern Germany still lag far behind those of western Germany, despite the overall strength of the German economy.

Finally, the "new economic geography" (see Fujita, et al.) has focused attention on the spatial dimension of the economy. In this emerging area of research, the geographic location of an economy may be even more significant than a national boundary. In fact, the new economic geography shows how economic disparities can surface even with equal resource endowments and in the absence of trade barriers. Since history plays an important role in the development of regional economies, these new research findings also suggest that economic policies may have a significant effect on local economic growth.

In light of this interest, regional policy analysis models can play an important role in evaluating the economic effects of alternative courses of action. Model users can answer "what if" questions about the economic effects of policies in areas such as economic development, energy, transportation, the environment, and taxation. Thus, simulation models for state, provincial, and local economies can help guide decision makers in formulating strategies for these geographical areas.

PI^+ (and its predecessor Policy Insight) is probably the most widely applied regional economic policy analysis model. Uses of the model to predict the regional economic and demographic effects of policies cover a range of issues; some examples include electric utility restructuring in Wyoming, the construction of a new baseball park for Boston, air pollution regulations in California, and the provision of tax incentives for business expansion in Michigan. The model is used by government agencies on the national, state, and local level, as well as by private consulting firms, utilities, and universities.

The original version of the model was developed as the Massachusetts Economic Policy Analysis (MEPA, Treyz, Friedlander, and Stevens) model in 1977. It was then extended into a model that could be generalized for all states and counties in the U.S. under a grant from the National Cooperative Highway

Research Program. In 1980, Regional Economic Models, Inc. (REMI) was founded to build, maintain, and advise on the use of the REMI model for individual regions. REMI was also established to further the theoretical framework, methodology, and estimation of the model through ongoing economic research and development.

Major extensions of the initial model include the incorporation of a dynamic capital stock adjustment process (Rickman, Shao, and Treyz, 1993), migration equations with detailed demographic structure (Greenwood, Hunt, Rickman, and Treyz, 1991; Treyz, Rickman, Hunt, and Greenwood, 1993), consumption equations (Treyz and Petraglia, 2001), and endogenous labor force participation rates (Treyz, Christopher, and Lou, 1996). A multi-regional national model has also been developed that has a central bank monetary response to economic changes that occur at the regional level (Treyz and Treyz, 1997).

Most recently, the model structure has been developed to include "new economic geography" assumptions. Economic geography theory explains regional and urban economies in terms of competing factors of dispersion and agglomeration. Producers and consumers are assumed to benefit from access to variety, which tends to concentrate production and the location of households. However, land is a finite resource, and high land prices and congestion tend to disperse economic activity.

Economic geography is incorporated in the model in two basic indexes. The first is the commodity access index, which predicts how productivity will be enhanced and costs reduced when firms increase access to intermediate inputs. This index is also used in the migration equation to incorporate the beneficial effect for consumers of having more access to consumer goods, which is factored into their migration decisions. The second index is the labor access index, which captures the favorable effect on labor productivity and thus labor costs when local firms have access to a wide variety of potential employees and are able to select employees whose skills best suit their needs.

II. Overview of the Model

PI+ is a structural economic forecasting and policy analysis model. It integrates input-output, computable general equilibrium, econometric, and economic geography methodologies. The model is dynamic, with forecasts and simulations generated on an annual basis and behavioral responses to compensation, price, and other economic factors.

The model consists of thousands of simultaneous equations with a structure that is relatively straightforward. The exact number of equations used varies depending on the extent of industry, demographic, demand, and other detail in the specific model being used. The overall structure of the model can be summarized in five major blocks: (1) Output and Demand, (2) Labor and Capital Demand, (3) Population and Labor Supply, (4) Compensation, Prices, and Costs, and (5) Market Shares. The blocks and their key interactions are shown in Figures 1 and 2.

Figure 1: REMI Model Linkages

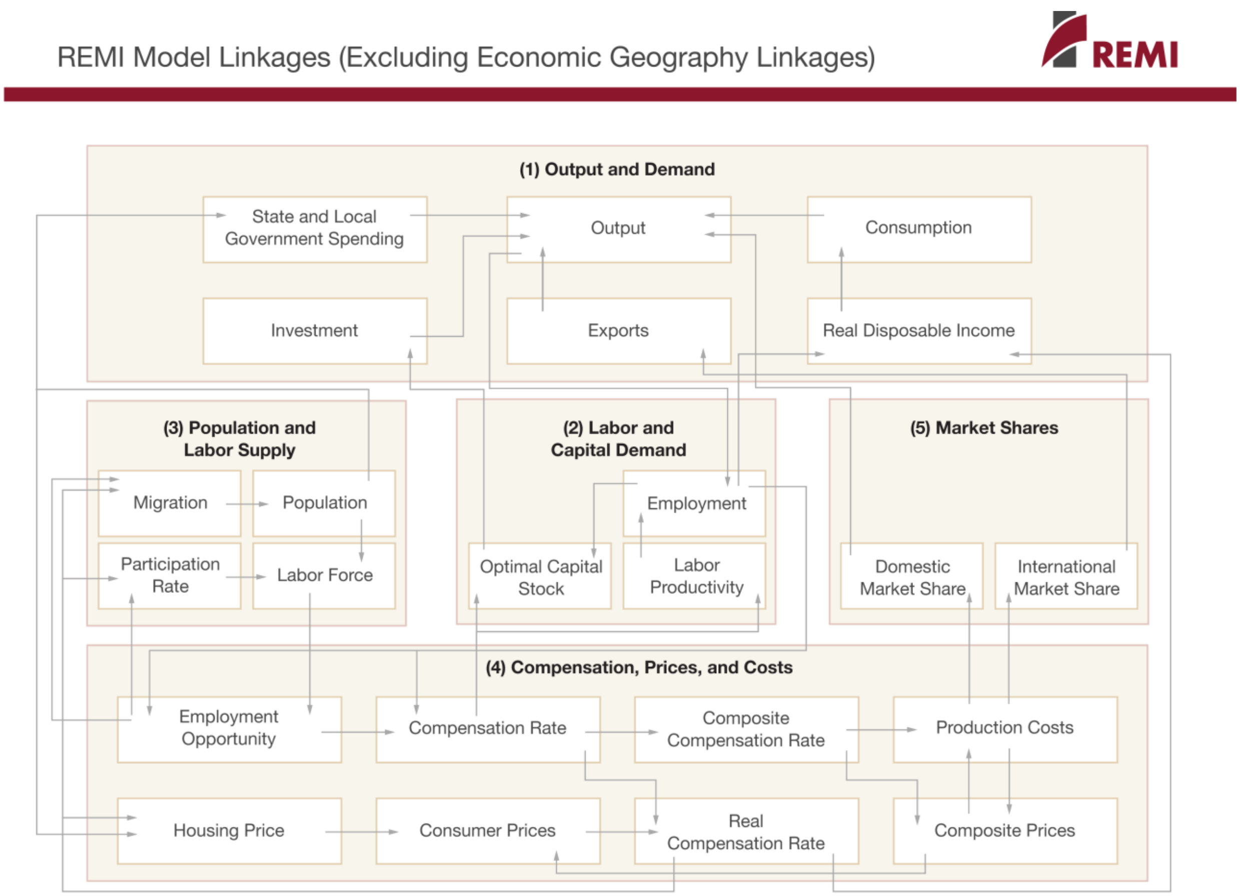
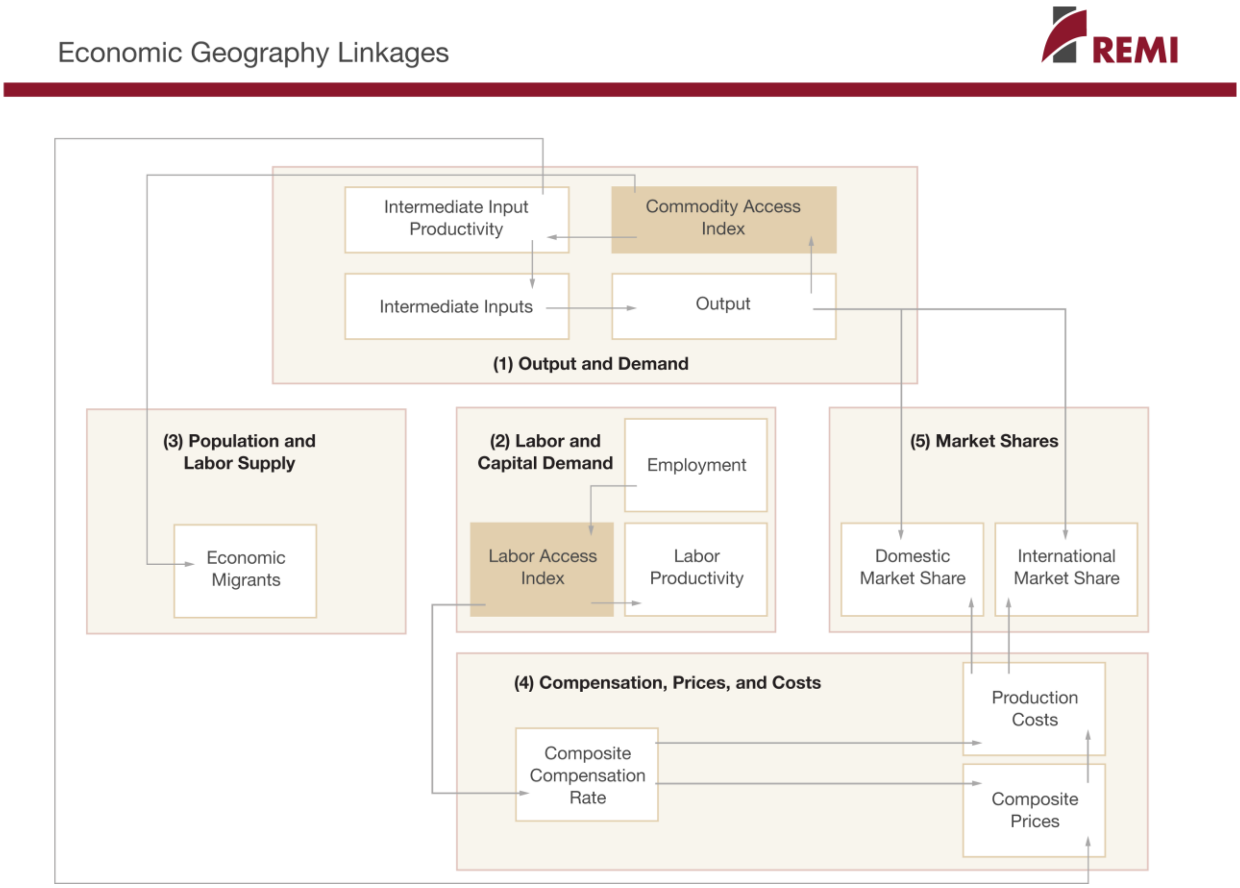


Figure 2: Economic Geography Linkages



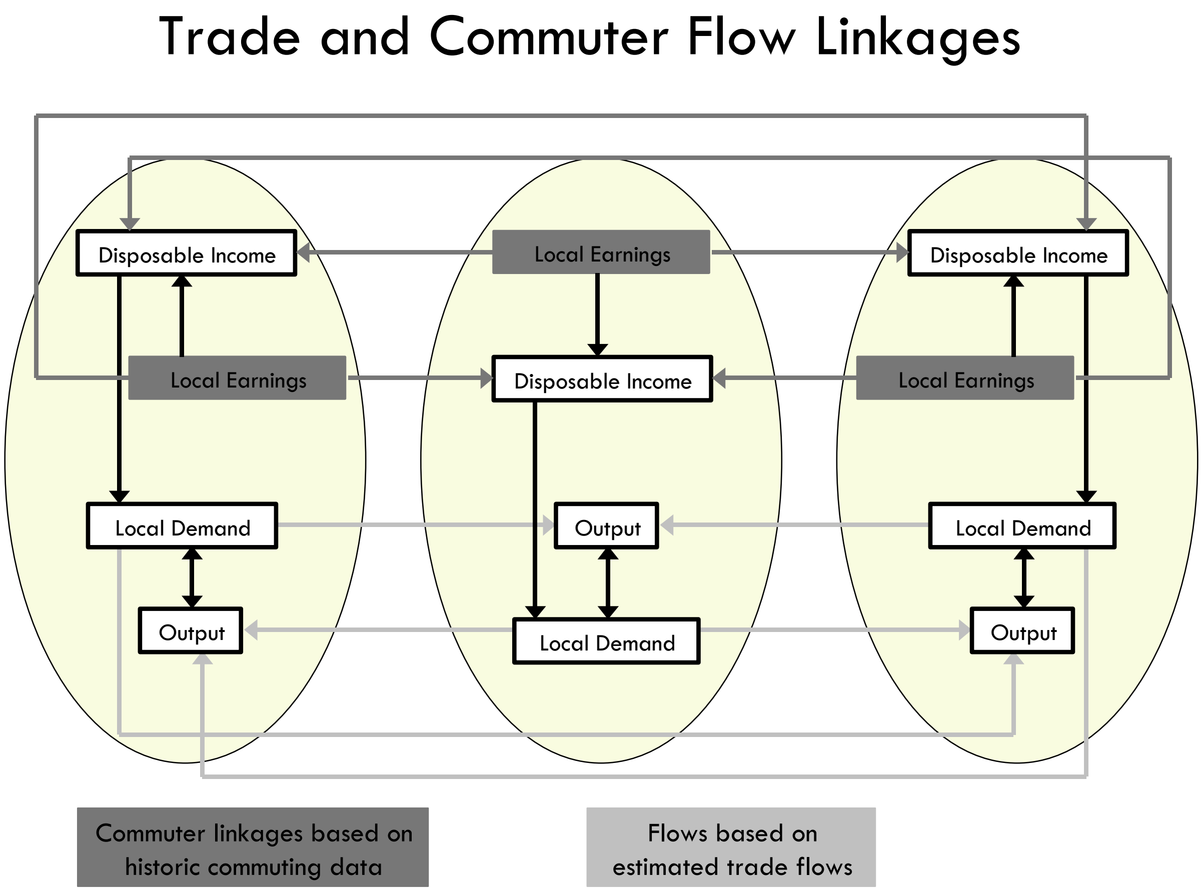
The Output and Demand block consists of output, demand, consumption, investment, government spending, exports, and imports, as well as feedback from output change due to the change in the productivity of intermediate inputs. The Labor and Capital Demand block includes labor intensity and productivity as well as demand for labor and capital. Labor force participation rate and migration equations are in the Population and Labor Supply block. The Compensation, Prices, and Costs block includes composite prices, determinants of production costs, the consumption price deflator, housing prices, and the compensation equations. The proportion of local, inter-regional, and export markets captured by each region is included in the Market Shares block.

Models can be built as single region, multi-region, or multi-region national models. A region is defined broadly as a sub-national area, and could consist of a state, province, county, or city, or any combination of sub-national areas.

Single-region models consist of an individual region, called the home region. The rest of the nation is also represented in the model. However, since the home region is only a small part of the total nation, the changes in the region do not have an endogenous effect on the variables in the rest of the nation.

Multi-regional models have interactions among regions, such as trade and commuting flows. These interactions include trade flows from each region to each of the other regions. These flows are illustrated for a three-region model in Figure 3. There are also multi-regional price and wage cost linkages as shown in the Figure at the end of Section III.

Figure 3: Trade and Commuter Flow Linkages



Multiregional national models also include a central bank monetary response that constrains labor markets. Models that only encompass a relatively small portion of a nation are not endogenously constrained by changes in exchange rates or monetary responses.

Block 1. Output and Demand

This block includes output, demand, consumption, investment, government spending, import, commodity access, and export concepts. Output for each industry in the home region is determined by industry demand in all regions in the nation, the home region's share of each market, and international exports from the region.

For each industry, demand is determined by the amount of output, consumption, investment, and capital demand on that industry. Consumption depends on real disposable income per capita, relative prices, differential income elasticities, and population. Input productivity depends on access to inputs because a larger choice set of inputs means it is more likely that the input with the specific characteristics required for the job will be found. In the capital stock adjustment process, investment occurs to fill the difference between optimal and actual capital stock for residential, non-residential, and equipment investment. Government spending changes are determined by changes in the population.

Block 2. Labor and Capital Demand

The Labor and Capital Demand block includes the determination of labor productivity, labor intensity, and the optimal capital stocks. Industry-specific labor productivity depends on the availability of workers with differentiated skills for the occupations used in each industry. The occupational labor supply and commuting costs determine firms' access to a specialized labor force.

Labor intensity is determined by the cost of labor relative to the other factor inputs, capital and fuel. Demand for capital is driven by the optimal capital stock equation for both non-residential capital and equipment. Optimal capital stock for each industry depends on the relative cost of labor and capital, and the employment weighted by capital use for each industry. Employment in private industries is determined by the value added and employment per unit of value added in each industry.

Block 3. Population and Labor Supply

The Population and Labor Supply block includes detailed demographic information about the region. Population data is given for age, gender, and ethnic category, with birth and survival rates for each group. The size and labor force participation rate of each group determines the labor supply. These participation rates respond to changes in employment relative to the potential labor force and to changes in the real after-tax compensation rate. Migration includes retirement, military, international, and economic migration. Economic migration is determined by the relative real after-tax compensation rate, relative employment opportunity, and consumer access to variety.

Block 4. Compensation, Prices and Costs

This block includes delivered prices, production costs, equipment cost, the consumption deflator, consumer prices, the price of housing, and the compensation equation. Economic geography concepts account for the productivity and price effects of access to specialized labor, goods, and services.

These prices measure the price of the industry output, taking into account the access to production locations. This access is important due to the specialization of production that takes place within each industry, and because transportation and transaction costs of distance are significant. Composite prices for each industry are then calculated based on the production costs of supplying regions, the effective distance to these regions, and the index of access to the variety of outputs in the industry relative to the access by other uses of the product.

The cost of production for each industry is determined by the cost of labor, capital, fuel, and intermediate inputs. Labor costs reflect a productivity adjustment to account for access to specialized labor, as well as underlying compensation rates. Capital costs include costs of non-residential structures and equipment, while fuel costs incorporate electricity, natural gas, and residual fuels.

The consumption deflator converts industry prices to prices for consumption commodities. For potential migrants, the consumer price is additionally calculated to include housing prices. Housing prices change from their initial level depending on changes in income and population density.

Compensation changes are due to changes in labor demand and supply conditions and changes in the national compensation rate. Changes in employment opportunities relative to the labor force and occupational demand change determine compensation rates by industry.

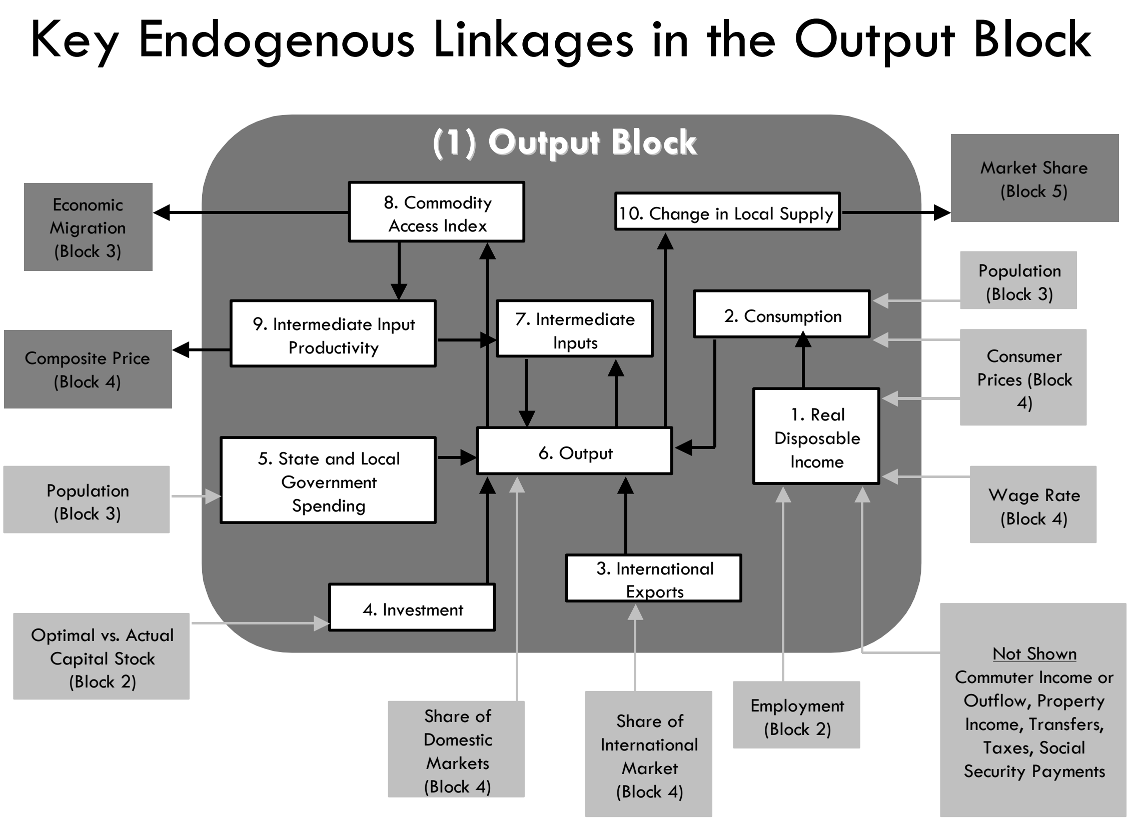
Block 5. Market Shares

The market shares equations measure the proportion of local and export markets that are captured by each industry. These depend on relative production costs, the estimated price elasticity of demand, and the effective distance between the home region and each of the other regions. The change in share of a specific area in any region depends on changes in its delivered price and the quantity it produces compared with the same factors for competitors in that market. The share of local and external markets then drives the exports from and imports to the home economy.

III. Detailed Diagrammatic and Verbal Description

The first task in this section is to examine the internal interactions within each of the blocks. The second task is to examine the linkages between the blocks. Finally, the last task is to tie it all together by looking at the key inter-block and intra-block linkages.

Block 1. Output and Demand



This block incorporates the regional product accounts. It includes output, demand, consumption, government spending, imports, and exports. The commodity access index, an economic geography concept, determines the productivity of intermediate inputs. Inter-industry transactions from the input-output table are also accounted for in this block.

Output for each industry in the home region is determined by industry demand in all regions in the nation, the home region's share of each market, and international exports from the region. The shares of home and other regions' markets are determined by economic geography methods, explained in block 5.

Consumption, investment, government spending, and intermediate inputs are the sources of demand. Consumption depends on real disposable income per capita, relative prices, the income elasticity of demand, and population. Consumption for all goods and services increases proportionally with population. The consumption response to per capita income is divided into high and low elasticity consumption components. For example, the demand for consumer goods such as vehicles, computers, and furniture is highly responsive to income changes, while health services and tobacco have low income elasticities. Demand for individual consumption commodities are also affected by relative prices. Changes in demand by consumption components are converted into industry demand changes by taking the proportion of each commodity for each industry in a bridge matrix.

Real disposable income, which drives consumption, is determined by compensation, employment, non-compensation income, and the personal consumption expenditure price index. Labor income depends on employment and the compensation rate, described in blocks 2 and 4, respectively. Non-compensation income includes commuter income, property income, transfers, taxes, and social security payments. Disposable income is stated in real terms by dividing by the consumer price index.

Investment occurs through the capital stock adjustment process. The stock adjustment process assumes that investment occurs in order to fill the gap between the optimal and actual level of capital. The investment in new housing, commercial and industrial buildings, and equipment is an important engine of economic development. New investment provides a strong feedback mechanism for further growth, since investment represents immediate demand for buildings and equipment that are to be used over a long period of time. The need for new construction begets further economic expansion as inputs into construction, especially additional employment in this industry, create new demand in the economy.

Investment is separated into residential, nonresidential, and equipment investment categories. In each case, the level of existing capital is calculated by starting with a base year estimate of capital stock, to which investment is added and depreciation is subtracted for each year. The desired level of capital is calculated in the capital demand equations, in block 2. Investment occurs when the optimal level of capital is higher than the actual level of capital; the rate at which this investment occurs is determined by the speed of adjustment.

Government spending at the regional and local level is primarily for the purpose of providing people with services such as schooling and police protection. However, government spending is usually linked to revenue sources. Thus, changes in government spending are driven by changes in population as well as the overall size of the economy (GRP). The government spending equation takes into account regional differences in per capita and per GDP government spending, as well as differential government spending levels across localities within a larger region.

The demand for intermediate inputs depends on the requirements of industries that use inputs from other sectors. These inter-industry relationships are based on the input-output table for the economy. For example, a region with a large automobile assembly plant would have a correspondingly large demand for primary metals, since this industry is a major supplier to the motor vehicles industry.

Thousands of specialized parts are needed to assemble an automobile, and the close proximity of the parts suppliers to the assembly plant is particularly significant under just-in-time inventory management procedures. More generally, the location of intermediate suppliers is important to at least some extent for every industry. Thus, the economic geography of the producer and input suppliers is a key aspect of regional productivity.

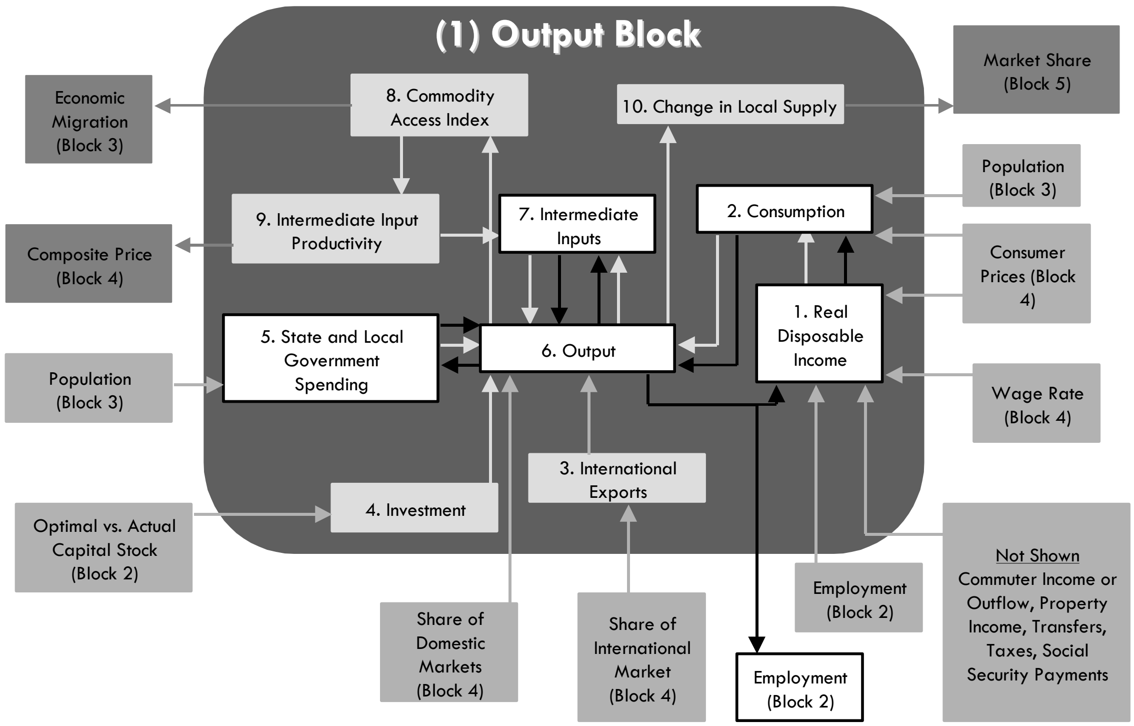
The agglomeration economies provided by the proximity of producers and suppliers is measured in the commodity access index. This index determines intermediate input productivity. The commodity access index for each industry is determined by the use of intermediate inputs, the effective distance to the input suppliers, and a measure of the productivity advantage of specialization in intermediate inputs. This productivity advantage is the elasticity of substitution between varieties in the production function. Although producers may be able to find a substitute for the precise component or service that they desire, access to the most favorable input provides a productivity advantage. When substitution between varieties is inelastic, then the productivity benefit of access to inputs is high. Thus, agglomeration economies are strong for the production of electrical equipment, computers, and machinery, and other industries that require specialized types of inputs for which substitution is difficult.

An increase in the output of an industry provides a larger pool of goods and/or services from which to choose. Since firms incur some fixed cost to produce a new variety, this increased pool of goods and services represents an increased availability of varieties. Therefore, an increase in industry output leads to a greater supply of differentiated goods and services, which can in turn lead to higher productivity and increase output. This positive feedback between tightly related clusters of industries is one source of regional agglomeration.

Since standard input-output analysis is often used to predict the effect of a firm either moving into or out of an area, it is important to explain why the results of the input-output analysis is incomplete. The following diagrams and explanation give an overview of the differences and similarities between PI^+ and Standard Input-Output.

In the first diagram ("Factors Included in Standard Input-Output Models"), white boxes ▭ indicate the linkages that constitute most I-O models.

Factors Included in Standard Input-Output Models



Some input-output models differentiate consumption by average household spending rates based on average earnings by industry. REMI differentiates between changes in income per capita and income changes due to changes in population, and includes different income elasticities for purchases of different consumer products (e.g. the consumption type that includes cigarettes has a lower income elasticity than the type that includes motor vehicles). Also, most I-O models would not account for the inflow and outflow of commuter earnings.

Thus, the I-O model captures the inter-industry flows that occur as output changes (each extra dollar of steel used 3 cents of coke) and it has feedbacks to consumer spending that are generated by changes in workers' income. Since population migration changes are not modeled, feedbacks to state and local governments in terms of new demands for per capita services are not included. Investment spending to construct new residential housing and commercial buildings cannot be modeled in static input-output models, because it is a transitory process that will occur when the need for housing and new stores occurs due to higher incomes and population but will return towards the baseline construction activity once the number of new houses and stores has risen enough to meet the one-time permanent increase in demand.

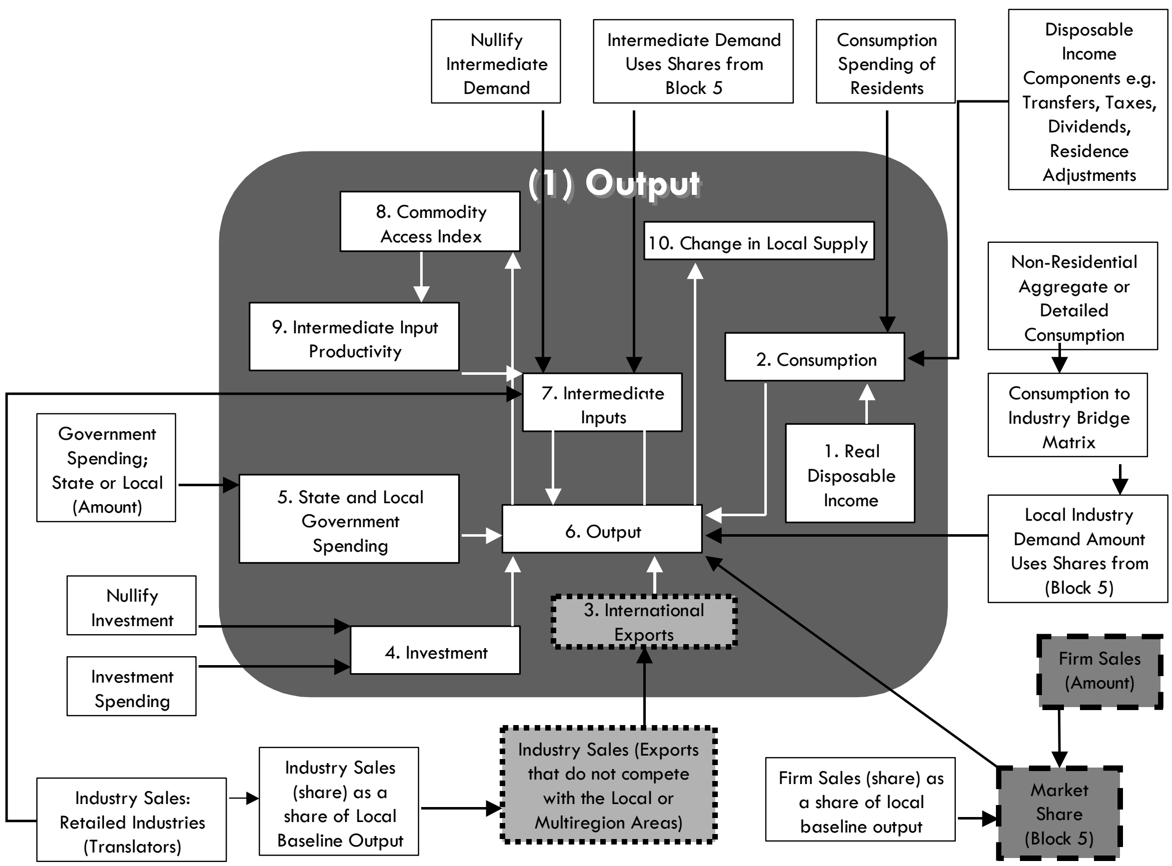
The change in the share of all markets as costs, the access to intermediate inputs, and the access to labor and feedback from other areas in a multi-region model are not included in standard I-O models. These all have effects in the short run, but the effects are even much larger in the long run. While an I-O analysis just gives a partial static picture, the REMI model catches all of the dynamic effects for each year in the future.

In addition to the difference in the extent of the important feedbacks in the REMI model compared to I-O, there is a major difference in the options for inputting policy variables in the two models. The following diagram shows the way standard input for the I-O model is Export Sales (going into International Exports) in comparison to the large number of inputs in the REMI model for Block 1.

REMI's Two Input Options vs. The Standard IO Single Option

Key Policy Variables for the Output and Demand Block

Block 1. Output and Demand



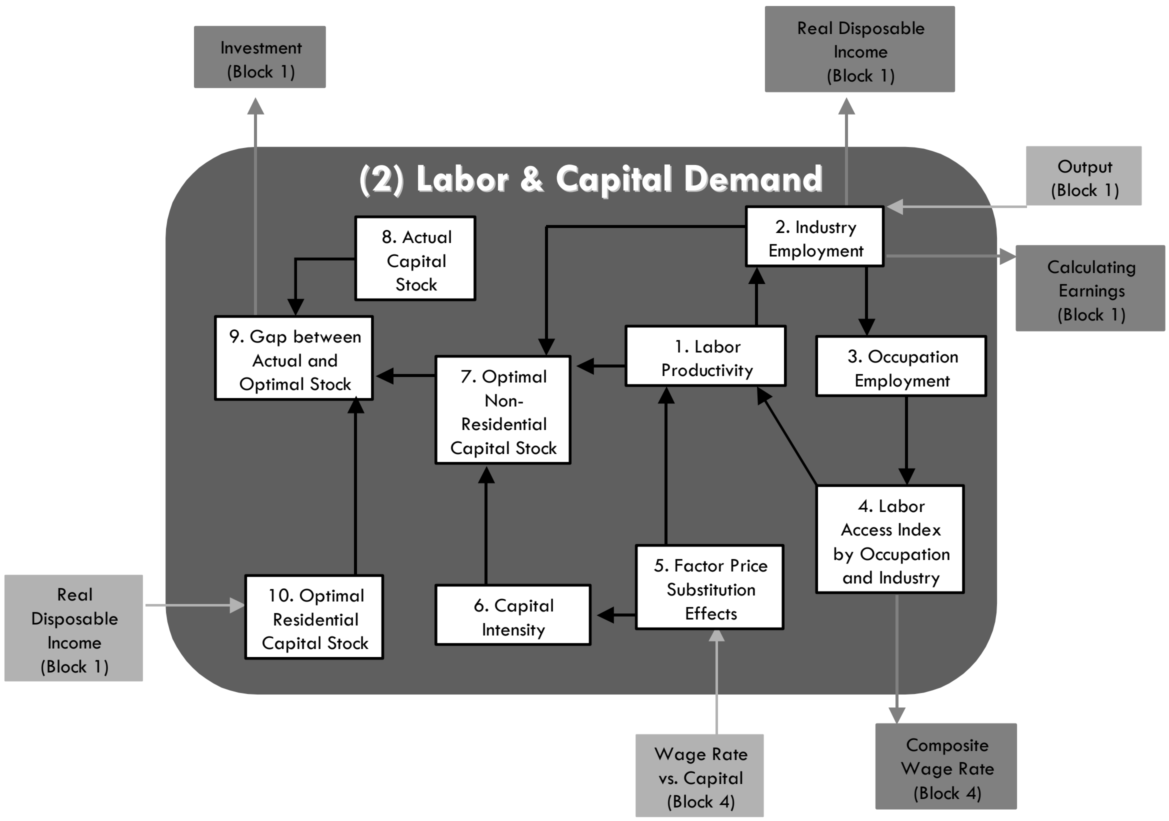
Standard input-output models only account for the direct output changes entered into the model, neglecting the displacement effects or augmenting effects on similar businesses in the region (or regions) modeled. The REMI model also provides this option.

Only the REMI model provides for inputting the output of the new firm in a way that accounts for displacement of competing employers in the home region and other regions in the multi-region model.

The alternative way that the REMI model provides for the effect of a firm entering or leaving a region due to a policy change can have substantial effects on the predicted outcome. For example, if a new grocery store is subsidized to move in, but 95% of all groceries are bought in the home region in the baseline case, then most of the sales of the new firm would displace sales in the grocery stores that are currently in the home region. This would mean that the net increase in jobs would only be a fraction of the firm's employment. The gain would mainly have to come from the increasing share in other regions, and this may be small if the initial shares indicate that the geographic area served by this industry is always very close to its source. In addition to considering the initial displacement, the REMI policy variable for a new firm will show how the future will be different if this new firm maintains its initial gain in share in the multi-region, the rest of the monetary union, and the rest of the world markets. Thus, the long-term effects will capture the differential effects of gaining share in an industry in which demand in the relevant markets is expanding rapidly versus those in which the demand is growing slowly. It will also capture the way that future projected changes in output per worker will mean that sales growth and employment growth may differ markedly.

The range of other policy variables for the output and demand block can be seen in the diagrams. These other ways that policy can influence the economic and demographic future of an area are not available for standard I-O models, because the linkages to most of the key processes that influence the outcomes in the region are not included in the structure of I-O models.

Block 2. Labor and Capital Demand



The Labor and Capital Demand block includes employment, capital demand, labor productivity, and the substitution among labor, capital, and fuel. Total employment is made up of farm, government, and private non-farm employment. Employment in private non-farm industries depends on employment demand and the number of workers needed to produce a unit of output. Employment demand is built up from the separate components of employment due to intermediate demand, consumer demand, local and regional government demand, local investment, and exports outside of the area. The employment per dollar of output depends on the national employment per dollar of output, the cost of other factors, and the access to specialized workers.

The availability of a large pool of workers within a region contributes to the labor force productivity. Each worker brings a set of unique characteristics and skills, even within the same occupational category. For example, a surgeon may specialize in heart, brain, or knee surgery. Although a brain surgeon may be able to perform a heart operation, the brain surgeon is likely to be less effective than a surgeon who has specific experience with heart surgery. Hospitals in major medical centers such as Houston are in an excellent position to meet their staff requirements because the number of qualified job applicants in the region is so large.

More broadly, locations that can be easily reached by a large number of potential employees can better match jobs with workers. The equation for labor productivity due to labor access is calculated separately for each occupation. Occupational productivity in each location is based on the residential location of all potential workers and their actual or potential commuting costs to that location.

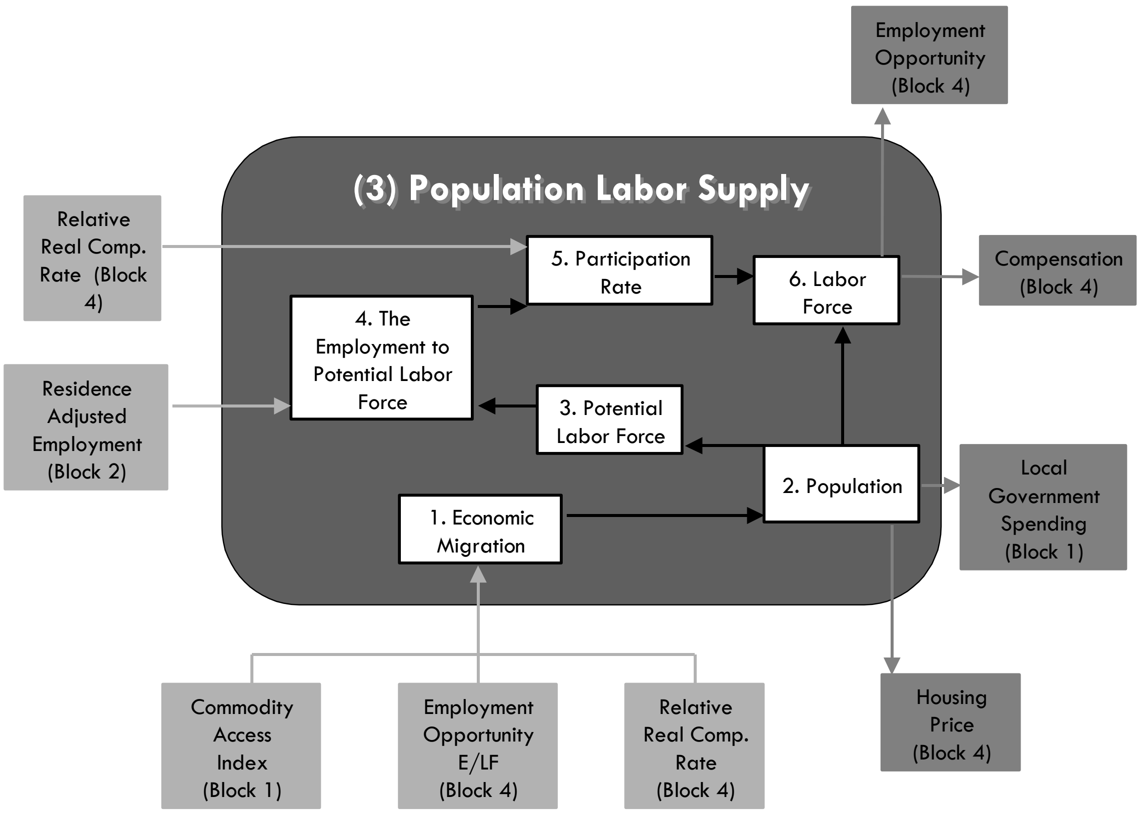
The contribution of labor variety to productivity is measured by an occupation-specific elasticity of substitution based on a study that considered wages and commuting patterns across a large metropolitan area. While the match of workers in specialized roles that are consistent with their training has a large impact on productivity for medical occupations, it is significantly less important for workers in the food service sector. Industry productivity due to specialization is built up from occupational productivity, using the proportionate number of workers in each occupation that are employed by a given industry.

The number of employees needed per unit of output depends on the use of other factors of production as well as labor access issues. Labor intensity, which measures the use of labor relative to other factors, is determined by the cost of labor relative to the cost of capital and fuel. The substitution between labor, capital, and fuel is based on a Cobb-Douglas production function, which implies constant factor shares. Labor intensity is calculated for each industry.

Demand for capital is driven by the optimal capital stock equation for industries and for housing. The optimal level of capital is determined for non-residential structures and equipment for each industry. The regional optimal capital stock is based on the industry size measured in capital-weighted employment terms, the cost of capital relative to labor, and a measure of the optimal capital stock on the national level. The variable for employment weighted by capital use is determined by the capital weight, employment, and labor productivity. The capital weight is the ratio of industry capital to employment in the region compared to the capital to employment ratio for the nation. The national optimal capital stock is based on the investment in the nation, the actual capital stock, the speed of adjustment, and the depreciation rate.

The optimal level of capital for residential housing is determined by the real disposable income in the region relative to the nation, the optimal residential capital stock for the nation, and the price of housing. To account for the cost of fuel, the fuel components of production (coal mining, petroleum refining, electric and natural gas utilities) are taken out of intermediate industry transactions and considered as a value-added factor of production. Then, firms substitute between labor, capital, and fuel (electric, natural gas, and residual fuel) as the relative costs of factor inputs change.

Block 3. Population and Labor Supply



The Population and Labor Supply block includes detailed demographic information about the region. The population is central to the regional economy, both as a source of demand for consumer and government spending and as the determinant of labor supply. As the composition of the population changes through births, deaths, and migration, so goes the region.

The demographic block is based on the cohort-component method. Population in any given year is determined by adding the net natural change and the migration change to the previous year's population. The natural change is caused by births and deaths, while migration occurs for economic and non-economic reasons. Population data is given for age, gender, and ethnic category.

Fertility rates are the ratio of births to the number of women in each age group. The survival rate is equal to one minus the death rate, which is the ratio of deaths to population in each cohort. Since fertility rates vary widely across age and ethnic groups, and survival rates vary widely for gender as well as age and ethnic category, the detailed demographic breakdown is needed to accurately capture the aggregate birth and survival rates.

Migration, economic or non-economic, also varies widely across population groups. Changes in retirement, international, and returning military migration are all assumed to occur for reasons that are not primarily due to with changing regional economic conditions. Retirement migration depends on the retirement-age population in the rest of the country for regions that have gained retirement population in the past, and on the retirement-age population within the regions for places that tend to have a net loss of retirees. The probability of losing or gaining a retiree is age and gender specific for each age group.

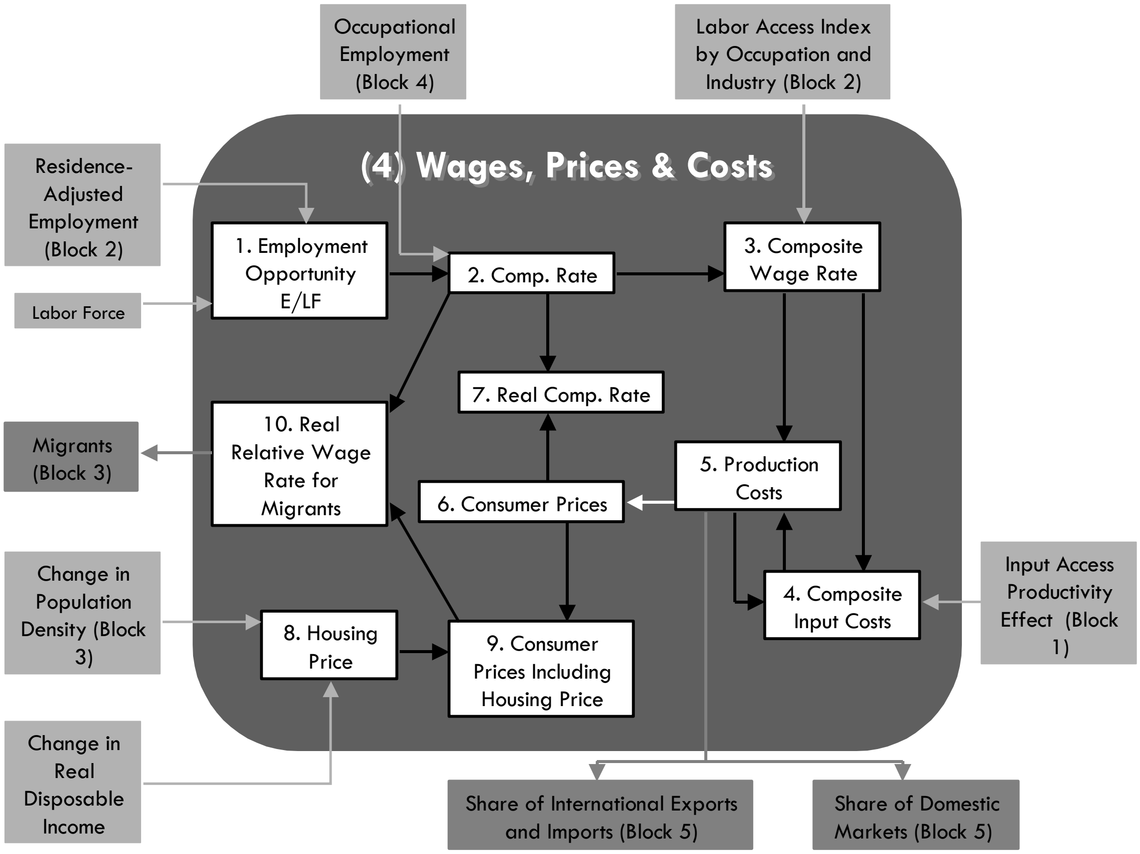
International migration is also based on previous patterns. Changes in political restrictions on immigration and the economy of the immigrants' country are more significant in determining international migration than are changes in the economy of the home region. Returning military migration patterns are also better explained by existing patterns than by regional economic conditions, so returning military is also an exogenous variable.

Economic migration is the movement of people to regions with better economic conditions. Economic migrants are attracted to places with relatively high wages and employment opportunities. Migrants are also attracted to places with high amenities. Potential migrants value access to consumer commodities, which depend on economic conditions. Thus, as the output of consumer goods and services increases, the amenity attraction of the region increases. Other amenities are due to non-economic factors. These amenities or compensating differentials are measured indirectly by looking at migration patterns over the last 10 years. In this way, the compensating differential is calculated as the expected compensation rate that would result in no net in- or out-migration. For example, people may be willing to work in Florida even if paid only 85% of the average U.S. compensation rate.

The labor force consists of unemployed individuals who are seeking work as well as employed workers. The labor force participation rate is thus the proportion of each population group that is working or looking for work. To predict the labor force, the model sums up the participation rate and cohort size for each demographic category. Participation rates vary widely across age, gender, and ethnic category; thus, the labor force depends in large part on the population structure of the region.

The willingness of individuals to participate in the labor force is also responsive to economic conditions. Higher compensation rates and greater employment opportunities generally encourage higher labor force participation rates. The extent to which rates change in response to these economic factors, however, differs substantially for different population groups. For example, the willingness of men to enter the labor force is more influenced by compensation, while women are more sensitive to employment opportunities.

Block 4. Compensation, Prices, and Costs

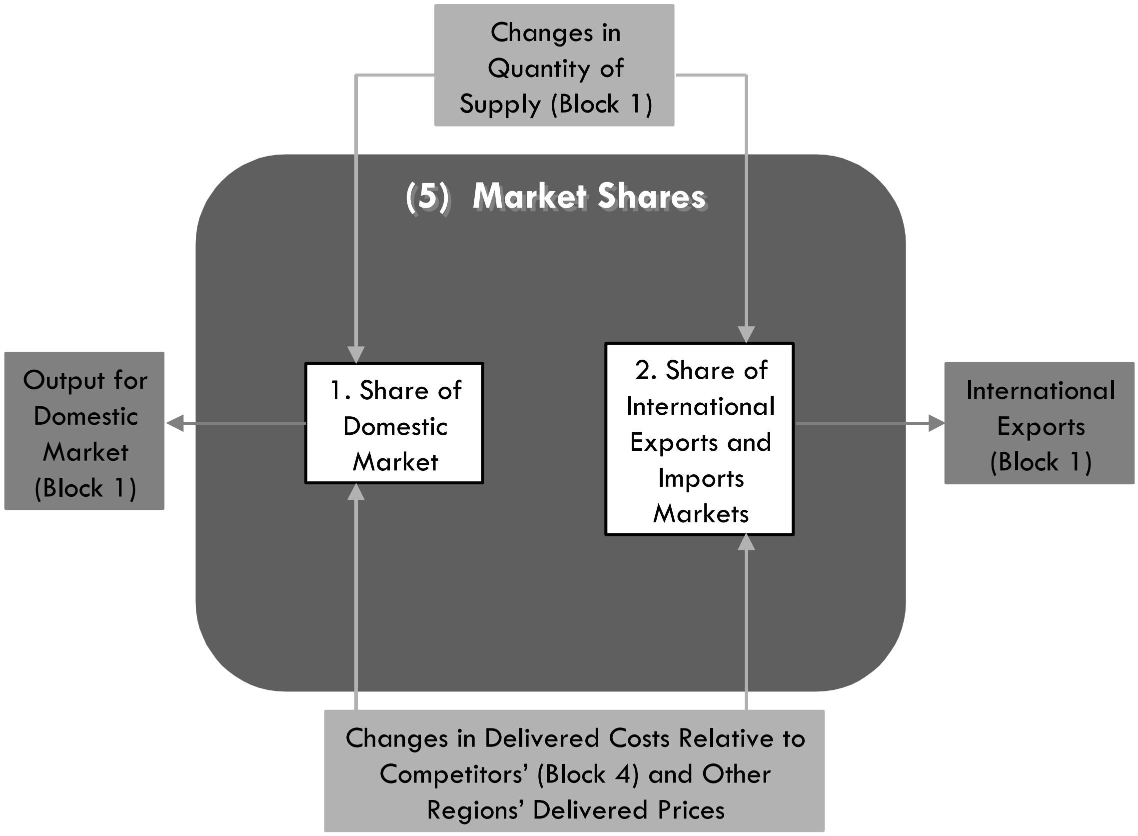


This block includes compensation, consumer prices, production costs, housing prices, and composite wages and input costs. Compensation, prices, and costs are determined by the labor and housing markets. The labor market is central to the regional economy, and compensation differences are the primary source of price and cost differentials between regions. Demand for labor, from block 2, and labor force supply, from block 3, interact to determine compensation rates. Housing prices depend on changes in population density and changes in real disposable income.

Economic geography concepts account for productivity and corresponding price effects due to access to specialized labor and inputs into production. The labor access index from block 2, as well as the nominal compensation rate, determines the composite compensation rate. The composite cost of production depends on the productivity-adjusted compensation rate of the region, costs of structures, equipment, and fuel, and the delivered price of intermediate inputs.

The delivered price of a good or service is based on the cost of the commodity at the place of origin, and the distance cost of providing the commodity to the place of destination. This price measure is calculated relative to delivered prices in all other regions, and weights the delivered price from all locations that ship to the home region.

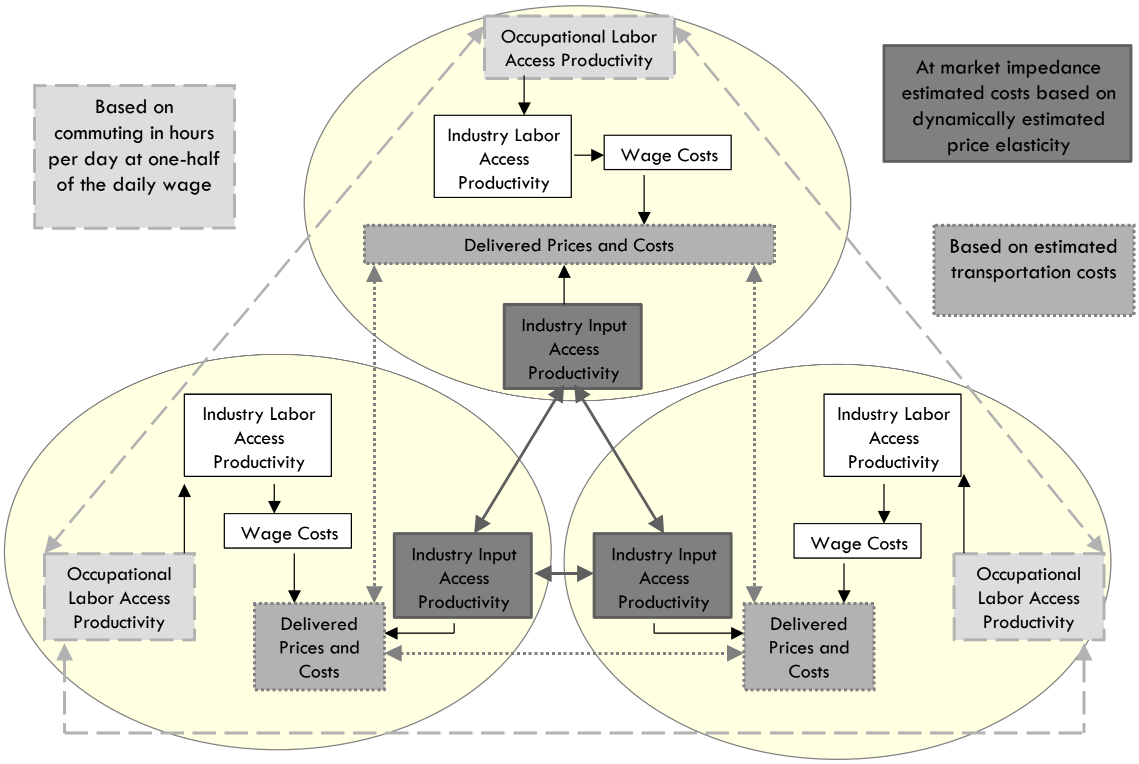
Block 5. Market Shares



The Market Shares block represents the ability of the region to sell its output within the local region, to other regions in the nation, and to other nations. Although the share of local markets is generally higher than any other market share, the equation for the market share of the home region is the same as for other regions within the nation. The share of international exports from the home region depends on national exports overall, and relative cost and output changes in the home region.

Changes in market shares within the nation depend on changes in industry production costs and output. Production cost increases lower market shares, but higher output raises market shares. Market shares rise with output increases, since higher output is better able to meet local and other regions' demand for goods and services by providing more choices.

Multi-Regional Price and Wage Linkages



IV. Block by Block Equations

Block 1 – Output and Demand

Output Equations

The output in region *k* for industry *i* is determined by the following equation:

$$Q_{i,t}^{k} = \sum_{l=1}^{m} s_{i,t}^{k,l}\, DD_{i,t}^{l} + sx_{i,t}^{k,row} * X_{i,t}^{u} + SALPOL6_{i,t}^{k} + SALPOL8_{i,t}^{k} + \left(\frac{EPOL4_{i,t}^{k}}{EPV_{i,T}^{k}}\right) \quad (1\text{-}1)$$

Where;

$Q_{i,t}^{k}$ =The output for industry *i* in region *k*.

$s_{i,t}^{k,l}$ = Region *k*'s share for industry *i* of the market in region *l*.
$DD_{i,t}^{l}$ =The domestic demand for industry *i* in region *l*.
$sx_{i,t}^{k,row}$ = Region *k*'s share of the national exports of *i* to the rest of the world (*row*).
$X_{i,t}^{u}$ =Exports of industry *i* from the nation (*u*) to the rest of the world.
m = The number of areas in the model (minimum 2). Also the letter that denotes the exogenous region (i.e. rest of the nation) for any model that does not incorporate a monetary feedback.
$SALPOL6_{i,t}^{k}$ = The policy variable for Industry Sales / Exogenous Production without Employment, Investment, and Compensation.
$SALPOL8_{i,t}^{k}$ = The additive policy variable for Industry Sales / Exogenous Production.
$EPOL4_{i,t}^{k}$ = The additive policy variable for Industry Employment / Exogenous Production without Output Demand Growth Based on Productivity Growth.
$EPV_{i,T}^{k}$ = Employees per dollar of output in industry *i*, time T, region *k*.

The $DD_{i,t}^{l}$ is the quantity demanded in region *l*. The $s_{i,t}^{k,l}$ term will incorporate the changes in region *k*'s share of industry *i* in region *l* that are due to the changes in *k*'s delivered price of *i* to *l* compared to the weighted average price charged by all of the areas that deliver to *l*, the variety of *i* offered in *k* compared with the variety offered by competitors in *l*, and the mix of fast-growing relative to slow-growing detailed industries that make up industry *i* in area *k* compared to the mix in the nation (see Block 5 below).

$$DD_{i,t}^{k} = \left[\left[\left(\sum_{j=1}^{ns}\left(\frac{a_{ij,t}^{u}}{MCPRODA_{i,t}^{k}}\right) * Q_{j,t}^{k} + \sum_{j=1}^{ncomm} a_{ij,t}^{u}\, C_{j,t}^{k} + \sum_{j=1}^{ninv} a_{ij,t}^{u}\, I_{j,t}^{k} + \sum_{j=1}^{ngov} a_{ij,t}^{u}\, G_{j,t}^{k}\right) + DEMPOL_{i,t}^{k}\right] * sd_{i,t}^{k}\right] - IMPPOL_{i,t}^{k} \quad (1\text{-}2)$$

Where;

$DD_{i,t}^{k}$ = The domestic demand for industry *i* in region *k*.
$a_{ij,t}^{u}$ = The average *i* purchased per dollar spent on *j* in the nation (*u*) in period *t*.
$MCPRODA_{i,t}^{k}$ = The moving average of $MCPROD_{i,t}^{k}$ (see below).

ns = The number of industries.
ncomm = The number of final demand consumption categories.
ninv = The number of investment categories.
ngov = The number of government categories.
$Q_{j,t}^{k}$ =The output for industry *j* in region *k*.

$C_{j,t}^{k}$ =The demand for consumption category *j* in region *k*.

$I_{j,t}^{k}$ =The demand for investment category *j* in region *k*.

$G_{j,t}^{k}$ =The demand for government category *j* in region *k*.

$sd_{i,t}^{k}$ = The share of area *k*'s demand for good *i* in time *t* that is supplied from within the nation.
$DEMPOL_{i,t}^{k}$ = The policy variable for Exogenous Final Demand.
$IMPPOL_{i,t}^{k}$ = The policy variable for Imports from Rest of World.

The commodity access index is determined by the change in the region's productivity of intermediate inputs due to changes in the access to these inputs.

$$MCPROD_{i,t}^{k} = \left[\frac{\left(\sum_{k=1}^{m} \left(\frac{Q_{i,t}^{k}}{\sum_{j=1}^{m} Q_{i,t}^{j}} \right) \left(\left(ED_{i}^{kj} \right)^{\eta_i} \right)^{(1-\sigma_i)} \right)^{\frac{1}{1-\sigma_i}}}{\left(\sum_{k=1}^{m} \left(\frac{Q_{i,T}^{k}}{\sum_{j=1}^{m} Q_{i,T}^{j}} \right) \left(\left(ED_{i}^{kj} \right)^{\eta_i} \right)^{(1-\sigma_i)} \right)^{\frac{1}{1-\sigma_i}}} \right]^{-1} * MCPRMPV_{i,t}^{k} \qquad (1\text{-}3)$$

Where;

$MCPROD_{i,t}^{k}$ = The commodity access (intermediate input) index. It predicts the change in the productivity of intermediate inputs due to changes in the access to these inputs in area *k*.
σ_i = The price elasticity of demand for industry *i*. (This parameter is estimated econometrically as the change in market share due to changes in an area delivered price compared to other competitors in each market in which an area sells products of industry *i*.)
ED_{i}^{kj} = The "effective distance" between *k* and *j*. (This variable is obtained by aggregating from the small area trade flows in our database.)
$Q_{i,t}^{k}$ =The output for industry *i* in region *k*.

η_i = Distance deterrence elasticity. This is estimated using the exponent in the gravity equation (β_i) and the estimated price elasticity σ_i and then using the identity $\eta_i = \frac{\beta_i}{\sigma_i - 1}$.
$MCPRMPV_{i,t}^{k}$ = The policy variable for Commodity Access Index.

$$MCPRODA_{i,t}^{k} = (1-\lambda) * MCPROD_{i,t}^{k} + \lambda MCPRODA_{i,t-1}^{k} \qquad (1\text{-}4)$$

$MCPRODA_{i,t}^{k}$ = The moving average of $MCPROD_{i,t}^{k}$.

$\lambda = 0.8$ = speed of adjustment for moving average.

$$CPROD_{j,t}^{k} = \prod_{i=1}^{ns} \left(MCPRODA_{i,t}^{k}\right)^{PCE_{i,j}^{u}} \qquad (1\text{-}5)$$

$CPROD_{j,t}^{k}$ = The consumption commodity *j* access index in region *k*.

$PCE_{i,j}^{u}$ = The proportion of each industry's input to consumption commodity *j*.

ns = The number of industries.

$$MIGPROD_{t}^{k} = \left(\prod_{j=1}^{ncomm} \left(\frac{CPROD_{j,t}^{k}}{CPROD_{j,t-1}^{k}}\right)^{WC_{j,t-1}^{u}}\right) * MIGPROD_{t-1}^{k} \qquad (1\text{-}6)$$

$MIGPROD_{t}^{k}$ = The consumer access index.

$MIGPROD_{T}^{k} = 1$

ncomm = The number of consumption categories.

$wc_{j,t-1}^{u}$ = Commodity *j*'s proportion of total national consumption in period *t-1*.

$$wc_{j,t-1}^{u} = \frac{C_{j,t-1}^{u}}{\sum_{j=1}^{ncomm} C_{j,t-1}^{u}}$$

Consumption Equations

The following consumption equation is used, which substitutes for the equation published in a 2001 article by George Treyz and Lisa Petraglia.[1]

$C_{j,t}^{k}$ = 1 [calibration effect] * 2 [age composition effect] * 3 [regional effect] * 4 [marginal income effect] * 5 [region-specific marginal price effect] * 6 [national consumption per capita effect] * 7 [local population]

(1). Calibration Effect — *(2). Age Composition Effect* — *(3). Regional Effect* — *(4). Marginal Income Effect* — *(5). Region-Specific Marginal Price Effect* — *(6). U.S. Forecast Effect* — *(7). Local Population*

$$C_{j,t}^{k} = \left\{ \left[\frac{\frac{YD_{T}^{k}}{N_{T}^{k}}}{\frac{YD_{T}^{u}}{N_{T}^{u}}}\right] * \left[\frac{\sum_{l=1}^{7}\left(\%DG_{l,t}^{k} * PC_{l,j}^{u}\right)}{\sum_{l=1}^{7}\left(\%DG_{l,t}^{u} * PC_{l,j}^{u}\right)}\right] * \left[\frac{\frac{c_{j,2012}^{-R}}{c_{j,2012}^{-u}}}{AgeCompEffect(2)}\right] * \left[\frac{\left(\frac{\frac{RYD_{t}^{k}+FDPVR_{t}^{k}}{N_{t}^{k}}}{\frac{RYD_{t}^{u}}{N_{t}^{u}}}\right)}{\left(\frac{\frac{RYD_{t}^{k}}{N_{t}^{k}}}{\frac{RYD_{T}^{u}}{N_{T}^{u}}}\right)}\right]^{\beta_j} * \left[\frac{\left(\frac{\frac{CIFP_{j,t}^{k} * CPPV_{j,t}^{k}}{P_{t}^{-k}}}{\frac{P_{j,t}^{u}}{P_{t}^{-u}}}\right)}{\left(\frac{\frac{P_{j,T}^{k}}{P_{T}^{-k}}}{\frac{P_{j,T}^{u}}{P_{T}^{-u}}}\right)}\right]^{Y_j} * \left(\frac{C_{j,t}^{u}}{N_{t}^{u}}\right) * N_{t}^{k} \right\} + FDPVC_{j,t}^{k}$$

(1-7)

Where;

[1] *Consumption Equations for a Multiregional Forecasting and Policy Analysis Model;* G.I. Treyz and L.M. Petraglia; *Regional Science Perspectives in Economic Analysis*, Elsevier Science B.V. 287-300; 2001.

$C_{j,t}^{k}$ =The demand for consumption category *j* in region *k*.

YD_{T}^{k} = Nominal Disposable Income in region *k* for the last history year (*T*).

N_{T}^{k} = Population in region *k* for the last history year (*T*).

$\%DG_{l,t}^{k}$ = percentage of demographic age group *l*.

$PC_{l,j}^{u}$ = Propensity to consume for nation, age group *l*, commodity *j*.

$\bar{C}_{j,2012}^{R}$ = Average consumption per household for commodity j, major region R (Northeast, Midwest, South, West), in t=2012.

RYD_{t}^{k} = Real Disposable Income in region *k,* time period *t.*

N_{t}^{k} = Population in region *k,* time period *t.*

$CIFP_{j,t}^{k}$ =The delivered price for consumption category *j* in region *k*.

$\bar{P}_{t}^{k}$ = Average price (weighted average of all the commodities that make up total consumptions) in region *k*.

β_{j} = Marginal income elasticities (estimated separately for luxuries and necessities)

Y_{j} = Marginal price elasticities (estimated separately for luxuries and necessities)

$FDPVR_{t}^{k}$ = The policy variable for Consumption Reallocation.
$FDPVC_{j,t}^{k}$ = The policy variable for Consumer Spending.

Real Disposable Income Equations

Real disposable income *(RYD)* in the region equals personal income *(YP)* adjusted for taxes *(TAX)* and the PCE-Price Index, which represents the cost of living *(*$\bar{P}$*)*. Total personal income *(YP)* depends on compensation (*COMP)*, and proprietors' income *(YPI)*, property income *(YPROP)*, employee and self-employed contributions for government social insurance *(TWPER)*, employer contributions for government social insurance *(EGSI*), transfer payments *(V)*, and an adjustment to account for the difference between place-of-work and place-of-residence earnings *(RA)*.

$$RYD_{t}^{k} = \frac{\left(YP_{t}^{k} - TAX_{t}^{k}\right)}{\bar{P}_{t}^{k}}$$

$$YP_{t}^{k} = COMPT_{t}^{k} + YPIT_{t}^{k} + YPROP_{t}^{k} - TWPER_{t}^{k} - EGSI_{t}^{k} + V_{t}^{k} + RA_{t}^{k}$$

Total compensation, *COMPT*, is an aggregation of individual industry wages and salaries and supplements to wages and salaries. Thus,

$$COMPT_t^k = \sum_{i=1}^{ns}\left(E_{i,t}^k * CR_{i,t}^k + WBPVA_{i,t}^k + WSDAPV2_{i,t}^k\right) \tag{1-8}$$

Where;

$COMPT_t^k =$ Total compensation aggregated across all industries.
$E_{i,t}^k =$ Employment in industry *i*.
$CR_{i,t}^k =$ The compensation rate of industry *i*.
$WBPVA_{i,t}^k =$ The policy variable for Wage and Salary Disbursements.
$WSDAPV2_{i,t}^k =$ The policy variable for Compensation.

The self-employed generate proprietors' income,

$$YPI_{i,t}^k = YLP_{i,t}^k - COMP_{i,t}^k + YPIPVA_{i,t}^k \tag{1-9}$$

Where;

$YPI_{i,t}^k =$ Proprietors' income for industry *i*.
$YLP_{i,t}^k =$ Labor and proprietors' income for industry *i*.
$COMP_{i,t}^k =$ Compensation for industry *i*.
$YPIPVA_{i,t}^k =$ The policy variable for Proprietors' Income.

Total labor and proprietors' income, *YLP*, (also referred to as earnings by place of work) for all industries in the region can be calculated as

$$YLPT_t^k = \sum_{i=1}^{ns}\left(E_{i,t}^k * ER_{i,t}^k + WBPVA_{i,t}^k + WSDAPV2_{i,t}^k\right) \tag{1-10}$$

Where;

$YLPT_t^k =$ Total labor and proprietors' income aggregated across all industries.
$E_{i,t}^k =$ Employment in industry *i*.
$ER_{i,t}^k =$ The earnings rate of industry *i*.
$WBPVA_{i,t}^k =$ The policy variable for Wage and Salary Disbursements.
$WSDAPV2_{i,t}^k =$ The policy variable for Compensation.

Wage and salary disbursements, *WSD*, are predicted as

$$WSDT_t^k = \sum_{i=1}^{ns}\left(E_{i,t}^k * WR_{i,t}^k + WBPVA_{i,t}^k + WSDAPV2_{i,t}^k\right) \tag{1-11}$$

Where;

$WSDT_t^k =$ Total wage and salary disbursements aggregated across all industries.
$E_{i,t}^k =$ Employment in industry *i*.
$WR_{i,t}^k =$ The wage rate of industry *i*.
$WBPVA_{i,t}^k =$ The policy variable for Wage and Salary Disbursements.

Property income, *YPROP*, is split into its major components of Dividends *(YDIV)*, Interest *(YINT)*, and Rent *(YRENT)*, which each depend on the population and its age distribution, as well as historical regional differences in the type of property income received.

$$YDIV_t^k = \lambda_{DIV,T}^k * NP_{DIV,t}^k * \left(\frac{YDIV_t^u}{NP_{DIV,t}^u}\right) + YPRPOL_{j,t}^k \quad \text{(1-12a)}$$

$$YINT_t^k = \lambda_{INT,T}^k * NP_{INT,t}^k * \left(\frac{YINT_t^u}{NP_{INT,t}^u}\right) + YPRPOL_{j,t}^k \quad \text{(1-12b)}$$

$$YRENT_t^k = \lambda_{RENT,T}^k * NP_{RENT,t}^k * \left(\frac{YRENT_t^u}{NP_{RENT,t}^u}\right) + YPRPOL_{j,t}^k \quad \text{(1-12c)}$$

$$YPROP_t^k = YDIV_t^k + YINT_t^k + YRENT_t^k \quad \text{(1-12d)}$$

$YDIV_t^k$ = Dividend income in region *k* for year *t*.
$\lambda_{DIV,T}^k$ = Adjustment for regional differences in dividend income based on the last history year.
$NP_{DIV,t}^k$ = Age-weighted population in region *k* for year *t*.
$YINT_t^k$ = Interest income in region *k* for year *t*.
$\lambda_{INT,T}^k$ = Adjustment for regional differences in interest income based on the last history year.
$NP_{INT,t}^k$ = Age-weighted population in region *k* for year *t*.
$YRENT_t^k$ = Rental income in region *k* for year *t*.
$\lambda_{RENT,T}^k$ = Adjustment for regional differences in rental income based on the last history year.
$NP_{RENT,t}^k$ = Age-weighted population in region *k* for year *t*.
$YPROP_t^k$ = Total property income in region *k* for year *t*.
$YPRPOL_{j,t}^k$ = The policy variable for each type of Property Income.

and

$$NP_{j,t}^k = L65_t^k + m65_j^u * G65_t^k \quad \text{(1-13)}$$

Where *m65* is the national ratio of per capita property income received (by type) for persons 65 years and older *(G65)* relative to property income received (by type) by persons younger than 65 *(L65)*, and $\lambda_{j,T}^k$ adjusts for regional differences and is calculated in the last historical year by solving equations (1-12) and (1-13).

Employee and self-employed contributions for government social insurance, *TWPER*, are predicted as

$$TWPER_t^k = \lambda_{TWPER,T}^k * WSDT_t^k * \left(\frac{TWPER_t^u}{WSDT_t^u}\right) + TWPPOL_t^k \quad \text{(1-14)}$$

Where $\lambda_{TWPER,T}^k$ is a coefficient calculated in the last historical year to adjust for regional differences in the *TWPER* per dollar of wage and salary disbursements, and *WSDT* equals total wage and salary disbursements.

$TWPPOL_t^k =$ The policy variable for Employee and Self-Employed Contributions for Government Social Insurance.

Employer contributions for government social insurance, *EGSI*, are predicted as

$$EGSI_t^k = \lambda_{EGSI,T}^k * WSDT_t^k * \left(\frac{EGSI_t^u}{WSDT_t^u}\right) + EGSIPVA_t^k \tag{1-15}$$

Where;

$\lambda_{EGSI,T}^k =$ a coefficient calculated in the last historical year to adjust for regional differences in the *EGSI* per dollar of wage and salary disbursements.

$EGSIPVA_t^k =$ The policy variable for Employer Contributions for Government Social Insurance.

The residence adjustment, *RA*, is used to convert place-of-work income (compensation, proprietors' income, and contributions for government social insurance) to place-of-residence income. Residence adjustment is calculated as the net of the gross commuter flows in, *GI*, and the gross commuter flows out, *GO*.

$$RA_t^k = GI_t^k - GO_t^k + RAPOL_t^k \tag{1-16}$$

$RAPOL_t^k =$ The policy variable for Residence Adjustment.

$$rs_t^{k,l} = \frac{LF_t^l * \left[P_t^l * \frac{YP_t^l}{YD_t^l}\right]^{(1-\sigma)} * \left(D^{k,l}\right)^{-\beta}}{\sum_{k \neq l}^n LF_t^j * \left[P_t^j * \frac{YP_t^j}{YD_t^j}\right]^{(1-\sigma)} * \left(D^{k,j}\right)^{-\beta}} \tag{1-17}$$

$rs_t^{k,l}$ = the share of commuters who live in region *l* and work in region *k* in time period *t*.
LF_t^l= labor force in region l in time period *t*.
P_t^l= the consumer price index including housing price in region l in time period *t*.
YP_t^l= total personal income in region l in time period *t*.
YD_t^l= total disposable income in region l in time period *t*.
$D^{k,l}$= the commute distance from region l to region k.
σ= Sigma value, the estimated parameter for consumer price.
β= Beta value, the estimated parameter for distance decay.

$$CI_t^{k,l} = \left(\sum_{k \neq l}^n rs_t^{k,l} * \left(COMPT_t^k - COMP_t^{nFM,k} - TWPER_t^k - EGSI_t^k\right)\right) + CommuterIncome\_PV_t^{k,l} \tag{1-18}$$

$CI_t^{k,l} =$ The commuter income flow from commuters who live in region *l* and work in region *k* in time period *t*.

$CommuterIncome\_PV_t^{k,l} =$ The policy variable for commuter income flow from commuters who live in region *l* and work in region *k* in time period *t*.

$$GI_t^k = \sum_{k \neq l}^{n} CI_t^{l,k} + GROSSEARN\_PV_{IN,t}^k \tag{1-19}$$

GI_t^k = Gross inflow of commuter dollars for residents of region *k* who work in all other areas.

$$GO_t^k = \sum_{k \neq l}^{n} CI_t^{k,l} + GROSSEARN\_PV_{OUT,t}^k \tag{1-20}$$

GO_t^k = Gross outflow from region *k* to all other areas.

$GROSSEARN\_PV_{IN,t}^k$ = The policy variable for gross inflow of commuter earnings to region *k* from all other areas.

$GROSSEARN\_PV_{OUT,t}^k$ = The policy variable for gross outflow of commuter earnings from region *k* to all other areas.

Transfer payments by component, V_j, depend on the number of persons in each of three groups: persons 65 years and older, persons younger than 65 who are not working, and all persons who are not working. The components of transfer payments also are adjusted for historical regional differences.

$$V_{j,t}^k = \lambda_{j,T}^k * NV_{j,t}^k * \left(\frac{V_{j,t}^u}{NV_{j,t}^u}\right) + VTRANSPOL_{j,t}^k \tag{1-21a}$$

$$V_t^k = \sum_j V_{j,t}^k \tag{1-21b}$$

Where;

$VTRANSPOL_{j,t}^k$ = The additive policy variable for individual components of Transfer Payments.

and

$$NV_{j,t}^k = VG_m^u * G65_t^k + VL_m^u[L65_t^k - EMPD_t^k] + [N_t^k - EMPD_t^k] \tag{1-22}$$

Where *VG* are per capita transfer payments (by four major types) for persons 65 years and older relative to per capita transfer payments (by four major types) for all persons not working, *VL* are per capita transfer payments (by four major types) for persons younger than 65 who are not working, relative to per capita transfer payments for all persons not working (by four major types), $\lambda_{j,T}^k$ adjusts for regional differences and is calculated in the last historical year, and *EMPD* and *N* are, respectively, total employed (scaled from residence adjustment) and population in the region.

The variable *TAX* depends on net income after subtracting transfer income. It is adjusted for regional differences by λ_T^k and changes as national tax rates change.

$$TAX_t^k = \lambda_T^k * \left(YP_t^k - V_t^k\right) * \left[\frac{TAX_t^u}{\left(YP_t^u - V_t^u\right)}\right] + TPOL_t^k \tag{1-23}$$

Investment Equations

There are four types of fixed investment to be considered: residential, nonresidential, equipment, and intellectual property products. Change in business inventories is the other component of investment, and is based on the national change in inventories as a proportion of sales applied to the size of the local industry.

The way in which the optimal capital stock (K^*) is calculated for each structure investment category (residential and non-residential) is explained in the factor and intermediate demand section below. Introducing time explicitly into the model, we can write equations that apply for residential and nonresidential fixed capital.

$$I_{j,t}^{k} = \alpha_j \left[K_{j,t}^{*k} - \left(1 - dr_{j,t}^{u}\right) * K_{j,t-1}^{k} \right] \quad (1\text{-}24)$$

$$K_{j,t-1}^{k} = \left(1 - dr_{j,t-1}^{u}\right) * K_{j,t-2}^{k} + I_{j,t-1}^{k} \quad (1\text{-}25)$$

Using equation (1-24), the actual capital stock in equation (1-25) can be replaced with the sum of the surviving initial capital stock (K_0) and the surviving previous investment expenditures. The investment equation is

$$KG_{j,t}^{k} = K_{j,0}^{*k} - \underbrace{\left(K_{j,t}^{*k} * \prod_{i=1}^{t} \left(1 - dr_{j,i}^{u}\right) + \sum_{i=1}^{t-1} I_{j,i}^{k} * \prod_{i+1}^{t} \left(1 - dr_{j,i}^{u}\right) \right)}_{K_{j,t}^{k}} \quad (1\text{-}26a)$$

or

$$KG_{j,t}^{k} = K_{j,t}^{*k} - \left(K_{j,t}^{k} + CAPPOL_{j,t}^{k} \right)$$

$$KGA_{j,t}^{k} = (1 - \lambda) * KG_{j,t}^{k} + \lambda KGA_{j,t-1}^{k} \quad (1\text{-}26b)$$

$$I_{j,t}^{k} = \left(\alpha_j * KGA_{j,t}^{k} \right) + FDPVI_{j,t}^{k} \quad (1\text{-}27)$$

Where;

$KG_{j,t}^{k}$ = Gap between current year's optimal and actual capital stock.

$KGA_{j,t}^{k}$ = Moving average of gap between optimal and actual capital stock for current year.

$KGA_{j,t-1}^{k}$ = Moving average of gap between optimal and actual capital stock for previous year.

$I_{j,t}^{k}$ = Investment demand for investment type *j*, time *t*, region *k*.

$K_{j,t}^{*k}$ = Optimal capital stock, type *j*, time *t*, region *k*.

$K_{j,0}^{*k}$ = Capital stock, type *j*, time 0, region *k*.

$dr_{j,i}^{u}$ = Depreciation rate, type *j*, time *t*.

a_j = Speed of adjustment, type *j*.

$\lambda = 0.5$ = speed of adjustment for moving average.

$CAPPOL_{j,t}^{k}$ = The variable for Capital Stock.

$FDPVI_{j,t}^{k}$ = The policy variable for components of Investment.

(For additional details see Rickman, Shao and Treyz, 1993).

Producers' durable equipment and Intellectual property products investments are calculated somewhat differently from residential and nonresidential investment. Since a very large part of these types of investment is for replacement, and not net new purchases, the following equation is used:

$$I_{J,t}^{k} = \left(1-\lambda_J\right)\left(\left(\frac{I_{NRS,t}^{k}}{I_{NRS,t}^{u}}\right) * I_{J,t}^{u}\right) + \lambda_J\left(\left(\frac{K_{NRS,t-1}^{k}}{K_{NRS,t-1}^{u}}\right) * I_{J,t}^{u}\right) + FDPVI_{J,t}^{k} \qquad (1\text{-}28)$$

$I_{J,t}^{k}$ = Investment demand for investment type J, time *t*, region *k*.

$I_{NRS,t}^{k}$ = Investment demand for nonresidential structures, time *t*, region *k*.

$K_{NRS,t-1}^{k}$ = Capital stock for nonresidential structures, time *t-1*, region *k*.

λ_J = Speed of adjustment for investment type J.

$FDPVI_{J,t}^{k}$ = The policy variable for components of Investment.

The national change in business inventories is allocated according to the regional share of employment.

$$CBI_{i,t}^{k} = \left(\frac{E_{i,t}^{k}}{E_{i,t}^{u}}\right) * CBI_{i,t}^{u} \qquad (1\text{-}29)$$

$CBI_{i,t}^{k}$ = The change in business inventories, industry *i*, region *k*.

$E_{i,t}^{k}$ = Employment, industry *i*, region *k*.

Government Spending Equations

The state and local government demand equations are driven based on the average per capita and per total value added demands for these services in the last history year (*T*).

$$G_{j,t}^{k} = \left[\left(\frac{TPNFVA\_PC\_A_t^{k}}{TPNFVA\_PC\_A_T^{k}}\right)^{\beta_j} * \frac{\frac{G_{j,t}^{u}}{N_t^{u}}}{\frac{G_{j,T}^{u}}{N_T^{u}}} * \frac{N_t^{k}}{N_T^{k}} * G_{j,T}^{k}\right] + FDPVSLG_{j,t}^{k} \qquad (1\text{-}30)$$

$$G_{j,T}^{k} = \lambda_j^{k} * N_T^{k} * \left(TPNFVA\_PC\_A_T^{k}\right)^{\beta_j} * \left(\frac{G_{j,T}^{u}}{N_T^{u}}\right) \qquad (1\text{-}31)$$

$$TPNFVA\_PC\_A_t^{k} = \lambda\left(TPNFVA\_PC\_A_{t-1}^{k}\right) + (1-\lambda)\left(\frac{\frac{TPNFVA_t^{k}}{N_t^{k}}}{\frac{TPNFVA_t^{u}}{N_t^{u}}}\right) \qquad (1\text{-}32)$$

Where;

$G_{j,t}^{k}$ = The demand for state or local government services (*j*) in region *k*, time *t*.

N_{t}^{k} = The total population in region *k*, time *t*.

$TPNFVA_{t}^{k}$ = The total private non-farm value added in region *k,* time *t*.

$TPNFVA\_PC\_A_{t}^{k}$ = The moving average of total private non-farm value added per capita in region *k* relative to the nation, time *t*.

λ_{j}^{k} = The local calibration factor for state or local government demand.

β_{j} = The elasticity of state or local government expenditures.

Superscript *u* indicates similar values for the nation.

Subscript *T* indicates similar values for the last history year.

λ = 0.5 = speed of adjustment for moving average.

$FDPVSLG_{j,t}^{k}$ = The policy variable for state or local government spending.

In the absence of adequate local demand estimates for state and local government separately, it is necessary to approximate these relative values based on assuming uniform productivity across all state and local government employees in the nation. It is important to note that local demand for local government services will be met in the local area, whereas the demand for state services in a local area may be met in part by state employees in the counties that provide state services, as set forth in the section on Market Shares below.

The federal civilian government demand equation is driven based on the region's share of national spending in the last history year (*T*).

$$G_{j,t}^{k} = \left[\frac{G_{j,T}^{k}}{G_{j,T}^{u}} * \ G_{j,t}^{u} \right] + \ FDPVFC_{t}^{k} \qquad (1\text{-}33)$$

$G_{j,t}^{k}$ = The demand for federal civilian government services (*j*) in region *k*, time *t*.

$FDPVFC_{t}^{k}$ = The policy variable for federal civilian government spending.

The federal military government demand equation is also driven based on the region's share of national spending in the last history year (*T*).

$$G_{j,t}^{k} = \left[\frac{G_{j,T}^{k}}{G_{j,T}^{u}} * \ G_{j,t}^{u} \right] + \ FDPVFM_{t}^{k} \qquad (1\text{-}34)$$

$G_{j,t}^{k}$ = The demand for federal military government services (*j*) in region *k*, time *t*.

$FDPVFM_{t}^{k}$ = The policy variable for federal military government spending.

Block 2 – Labor and Capital Demand

Labor Demand Equations

The productivity of labor depends on access to a labor pool. In this instance, we have chosen to use employment by occupation as the measure of access to the specialized labor pool. Thus, the variety effect on the productivity of labor by occupation is expressed in the following equation:

$$FLO_{j,t}^{k} = \frac{1}{\left(\sum_{l=1}^{m} \frac{EO_{j,t}^{l}+OTRPV_{j,t}^{l}}{EO_{j,t}^{u}} * \left(1+cc^{l,k}\right)^{1-\sigma_j}\right)^{\frac{1}{1-\sigma_j}}} \quad (2\text{-}1a)$$

$$RCW_{i,t}^{k} = \frac{1}{\left(\sum_{l=1}^{m} \frac{E_{i,t}^{l}}{E_{i,t}^{u}} * \left(1+cc^{l,k}\right)^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}}} \quad (2\text{-}1b)$$

$FLO_{j,t}^{k}$ = Labor productivity for occupation type *j* that depends on the relative access to labor in occupation *j* in region *k*, time *t*.

$RCW_{i,t}^{k}$ = Relative labor productivity due to industry concentration of labor.

$EO_{j,t}^{l}$ = Labor of occupation type *j* in region *l*, time *t*.

σ_j = Elasticity of substitution (i.e. cost elasticity).

$cc^{l,k}$ = Commuting time and expenses from *l* to *k* as a proportion of the wage rate.

$E_{i,t}^{l}$ = Employment in industry *i*, time *t*, in region *l*.

m = Number of regions in model including the rest of the nation region.

$OTRPV_{j,t}^{l}$ = The policy variable for Occupational Training.

The value of σ_j is based on elasticity estimates made by REMI under a grant from the National Cooperative Highway Research Program (Weisbrod, Vary, and Treyz, 2001) based on cross-commuting among workers in the same occupation observed in 1300 Traffic Analysis Zones in Chicago. Key data inputs on travel times were provided by Cambridge Systematics, Inc.

In order to determine labor productivity changes by industry due to access to variety, a staffing pattern matrix is used as follows:

$$FL_{i,t}^{k} = \left[\left(\frac{\left(\frac{\left(\sum_{j=1}^{nocc} d_{j,i} * FLO_{j,t}^{k}\right)+RCW_{i,t}^{k}}{2}\right)}{FL_{i,T}^{k}}\right)\right] * FLPRMPV_{i,t}^{k} \quad (2\text{-}1c)$$

$FL_{i,t}^{k}$ = Labor productivity due to labor access to industry and relevant occupations by industry *i*, in region *k*, time *t*, normalized by $FL_{i,T}^{k}$.

$d_{j,i}$ = Occupation *j*'s proportion of industry *i*'s employment.

$FLO_{j,t}^{k}$ = Labor productivity for occupation type *j* that depends on the relative access to labor in occupation *j* in region *k*, time *t*.

nocc = The number of occupations in industry *i*.

$FL_{i,T}^{k}$ = Labor productivity due to access by industry *i* in region *k* in the last year of history.

$RCW_{i,t}^{k}$ = Relative labor productivity due to industry concentration of labor.

$FLPRMPV_{i,t}^{k}$ = The policy variable for Labor Access Index.

Relative labor intensity is determined by the following equation based on Cobb-Douglas technology and the assumption that the optimal labor intensity is chosen when new equipment is installed.

$$L_{i,t}^{k} = L_{i,t-1}^{k} + \left(\frac{I_{NRS,t}^{k}}{K_{NRS,t}^{k}}\right) * \left[\underbrace{\left(RLC_{i,t}^{k}\right)^{b_{ji,t}^{u}-1}\left(RCC_{i,t}^{k} * COSCAP_{i,t}^{k}\right)^{b_{ji,t}^{u}}\left(\frac{FuelC_{i,t}^{k}}{FuelC_{i,t}^{u}}\right)^{b_{ji,t}^{u}}}_{h_{i,t}^{k}} - L_{i,t-1}^{k}\right] \quad (2\text{-}2)$$

$L_{i,t}^{k}$ = Relative labor intensity, industry *i*, time *t*, region *k*.

$b_{ji,t}^{u}$ = Contribution to value added of factor *j*, (labor, capital, and fuel respectively), industry *i*, time *t*.

$I_{NRS,t}^{k}$ = Nonresidential investment, region *k*, time *t*.

$K_{NRS,t}^{k}$ = Nonresidential capital stock, region *k*, time *t*.

$RLC_{i,t}^{k}$ = Relative labor cost, industry *i*, time *t*, region *k* equals $\left(\frac{CR_{i,t}^{k}}{CR_{i,t}^{u}}\right)$, before accounting for labor productivity effects.

$CR_{i,t}^{k}$ = The compensation rate of industry *i* in region *k*.

$CR_{i,t}^{u}$ = The compensation rate of industry *i* in the nation.

$RCC_{i,t}^{k}$ = Relative capital cost, industry *i*, time *t*, region *k*.

$COSCAP_{i,t}^{k}$ = The multiplicative policy variable for Capital Cost.

$FuelC_{i,t}^{k}$ = The weighted cost of fuel of industry *i* in region *k*.

$$FuelC_{i,t}^{k} = \left(\prod_{j=1}^{f}\left(RFuel_{j,jj,t}^{k} * RFCPV_{i,j,t}^{k}\right)^{FVW_{i,j,T}^{S}}\right)$$

$RFuel_{j,jj,t}^{k}$ = The relative cost of fuel by type and category in region *k*.

$RFCPV_{i,j,t}^{k}$ = The policy variable for Fuel Cost by industry *i* and type *j* in region *k*.

$FVW_{i,j,T}^{S}$ = The fuel expenditure weights for industry *i*, type *j*, and state *S* in the last history year.

$FuelC_{i,t}^{u}$ = The weighted cost of fuel of industry *i* in the nation.

$h_{i,t}^{k}$ = Optimal labor intensity, industry *i*, time *t*, region *k*.

Simplified, the above equation can be written as,

$$L_{i,t}^{k} = L_{i,t-1}^{k} + \left(\frac{I_{NRS,t}^{k}}{K_{NRS,t}^{k}}\right) * \left(h_{i,t}^{k} - L_{i,t-1}^{k}\right) \quad (2\text{-}3)$$

Where;

$$EPV_{i,t}^{k} = \frac{\frac{L_{i,t}^{k}}{L_{i,T}^{k}} * \left(\frac{E_{i,T}^{k}}{Q_{i,T}^{k}} * \frac{\frac{E_{i,t}^{u}}{Q_{i,t}^{u}}}{\frac{E_{i,T}^{u}}{Q_{i,T}^{u}}} \right) * \left(FLA_{i,t}^{k} \right)^{-b_{ij,t}^{u}} * epvindx_{i,t}}{\left(RPRDPV_{i,t}^{k} * \left(RLABPV_{i,t}^{k} \right)^{b_{ij,t}^{u}} \right)} \tag{2-4}$$

$EPV_{i,t}^{k}$ = Employees per dollar of output in industry *i*, time *t*, region *k*.

$L_{i,t}^{k}$ = Labor intensity due to relative factor costs, industry *i*, time *t*, region *k*.

$\frac{E_{i,t}^{u}}{Q_{i,t}^{u}}$ = Employees per dollar of output in the nation (*u*) in time *t*.

$b_{ij,t}^{u}$ = Labor share of industry *i* in time *t*.

$FLA_{i,t}^{k}$ = Moving average of labor productivity due to access by industry *i* in region *k*, time *t*, divided by $FL_{i,T}^{k}$.

$\frac{E_{i,T}^{u}}{Q_{i,T}^{u}}$ = Employees per dollar of output in the nation (*u*) in the last history year.

$\frac{E_{i,T}^{k}}{Q_{i,T}^{k}}$ = Employees per dollar of output in region *k* in the last history year.

$L_{i,T}^{k}$ = Labor intensity due to relative factor costs in industry *i*, region *k*, in the last history year.

$epvindx_{i,t}^{k}$ = Change in region *k*'s detailed industry mix relative to the nation since the last year of history (=1 if detailed industry national forecast is not used).

$RPRDPV_{i,t}^{k}$ = The policy variable for Factor Productivity.

$RLABPV_{i,t}^{k}$ = The policy variable for Labor Productivity.

Where;

If $YLP_{i,T}^{k} \geq COMP_{i,T}^{k}$ Then

$$Q_{i,T}^{k} = \frac{YLP_{i,T}^{k}}{YLP_{i,T}^{u}} * Q_{i,T}^{u} \tag{2-4a}$$

Otherwise

$$Q_{i,T}^{k} = \frac{COMP_{i,T}^{k}}{COMP_{i,T}^{u}} * Q_{i,T}^{u} \tag{2-4b}$$

In a multi-industry model, total employment in the area can be divided into three categories consisting of private non-farm industries, employment in the farm sector, and employment in government. Government is further divided into employment in state and local government sectors, and employment in federal civilian and military sectors. Output in private non-farm industries is determined by demand for inputs into the production process (intermediate demand) and demand from personal consumption, government, investment, and exports (final demand), and employees per unit of output (*EPV*). The equation for employment in private industry *i* for the single area model is

$$E_{i,t}^{k} = EPV_{i,t}^{k} * \left(QLI_{i,t}^{k} + QLC_{i,t}^{k} + QLG_{i,t}^{k} + QLINV_{i,t}^{k} + QXRMA_{i,t}^{k} + QXRON_{i,t}^{k} + QXROW_{i,t}^{k}\right) \quad (2\text{-}5)$$

Where;

$QLI_{i,t}^{k}\left(= \sum_j s_{i,t}^{k,k} * a_{ij,t}^{k} * Q_{j,t}^{k}\right)$ are sales of industry *i*'s product dependent on local intermediate demand.

$a_{ij,t}^{k} = \left(\frac{a_{ij,t}^{u}}{MCPRODA_{i,t}^{k}}\right)$ = The average *i* purchased per dollar spent on producing *j* in region *k* in time period *t*.

$QLC_{i,t}^{k}\left(= \sum_j s_{i,t}^{k,k} * a_{ij,t}^{u} * C_{j,t}^{k}\right)$ are sales dependent on local consumer demand.

$QLG_{i,t}^{k}\left(= \sum_j s_{i,t}^{k,k} * a_{ij,t}^{u} * G_{j,t}^{k}\right)$ are sales dependent on government demand.

$QLINV_{i,t}^{k}\left(= \sum_j s_{i,t}^{k,k} * a_{ij,t}^{u} * I_{j,t}^{k}\right)$ are sales dependent on local investment.

$QXRMA_{i,t}^{k}\left(= \sum_l s_{i,t}^{k,l} * DD_{i,t}^{l}\right)$ are sales to other areas in the in the multi-area model.

$QXRON_{i,t}^{k}\left(= s_{i,t}^{k,ron} * DD_{i,t}^{ron}\right)$ are sales to the rest of the nation.

$QXROW_{i,t}^{k}\left(= sx_{i,t}^{k,row} * X_{i,t}^{u}\right)$ are sales to the rest of the world.

Federal government employment in the local area is a fixed proportion of government employment in the nation, based on the last observed proportion. The equations for federal civilian employment and federal military employment are

$$EG_{fc,t}^{k} = \frac{EG_{fc,T}^{k}}{EG_{fc,T}^{u}} * EG_{fc,t}^{u} \quad (2\text{-}6)$$

$$EG_{fm,t}^{k} = \frac{EG_{fm,T}^{k}}{EG_{fm,T}^{u}} * EG_{fm,t}^{u} \quad (2\text{-}7)$$

Where;

$EG_{fc,t}^{k}$ = Federal civilian employment in area *k* in time *t* (where *T* is the last history year)

$EG_{fm,t}^{k}$ = Federal military employment in area *k* in time *t* (where *T* is the last history year)

u = As a superscript, denotes the nation.

State (EG_{st})and local government (EG_{loc})employment are based on estimated output per state or local government employee. In the absence of such regional data the national average is used as the ratio of state and local output to state and local government employment. Changes in per capita state and local government in the nation and changes in the population that is served by state and/or local government drive state and local employment. Thus, non-farm employment, *ENF*, is

$$ENF_t^k = \sum_{i=1}^{np} E_{i,t}^k + EG_{st,t}^k + EG_{loc,t}^k + EG_{fc,t}^k + EG_{fm,t}^k \quad (2\text{-}8)$$

Farm employment is estimated as a fixed share of national farm employment based on the last year of history. The equation for total employment (*ETOT*) is

$$ETOT_t^k = ENF_t^k + EF_t^k \quad (2\text{-}9)$$

Where;

EF is farm employment.

Capital Demand Equations

The optimal capital stock equation for non-residential structures (*j*=1) is:

$$K_{1,t}^{*k} = \left(\frac{\sum_{i=1}^{np} kw_i * RLC_{i,t}^k * UECPV_{i,t}^k}{\sum_{i=1}^{np} kw_i * RCC_{i,t}^k * COSCAP_{i,t}^k}\right) * \frac{AE_t^k}{AE_t^u} * K_{1,t}^{*u} * KP_1^k \quad (2\text{-}10)$$

$K_{1,t}^{*k}$ = Optimal capital stock for non-residential structures, time *t*, region *k*.

kw_i = Industry *i*'s share of total capital stock.

$RLC_{i,t}^k$ = Relative labor cost, industry *i*, time *t*, region *k* equals $\left(\frac{CR_{i,t}^k}{CR_{i,t}^u}\right)$, before accounting for labor productivity effects.

$CR_{i,t}^k$ = The compensation rate of industry *i* in region *k*.

$CR_{i,t}^u$ = The compensation rate of industry *i* in the nation.

$RCC_{i,t}^k$ = Relative capital cost, industry *i*, time *t*, region *k*.

$COSCAP_{i,t}^k$ = The multiplicative policy variable for Capital Cost.

AE_t^k = Employment weighted by capital use, time *t*, region *k* (used instead of employment because the variation in capital use per employee across industries is very large).

KP_1^k = Capital preference parameter for non-residential structures, region *k*.

$UECPV_{i,t}^k$ = The policy variable for Non-Compensation Labor Costs.

The term of $\sum kw_i * RLC_{i,t}^k$ (or $\sum kw_i * RCC_{i,t}^k$), in equation 2-10 above, is the average relative labor cost (or average relative capital cost) weighted by capital in use. The equation used to determine the variable *AE* is

$$AE_t^k = \sum_{i=1}^{np} kwe_i * \left(E_{i,t}^k - EPOL2_{i,t}^k - \left(SALPOL2_{i,t}^k * EPV_{i,t}^k\right)\right) * \left(FL_{i,t}^k\right)^{b_{ij,t}^u} \quad (2\text{-}11)$$

$kwe_i = \sum_{i=1}^{np} \frac{\frac{K_i^u}{TK^u}}{\frac{E_i^u}{TE^u}}$ = The average capital per employee in the nation.

$E_{i,t}^k$ = Employment in industry *i*, time *t*, region *k*.

$EPV_{i,t}^k$ = Employees per dollar of output in industry *i*, time *t*, region *k*.

$FL_{i,t}^{k}$ = Labor productivity due to labor access to industry and relevant occupations by industry *i*, in region *k*, time *t*, normalized by $FL_{i,T}^{k}$.

$b_{ij,t}^{u}$ = Labor share of industry *i* in time *t*.

$EPOL2_{i,t}^{k}$ = The policy variable for Nullify Investment Induced by Employment (Industry Sales).

$SALPOL2_{i,t}^{k}$ = The policy variable for Nullify Investment Induced by Industry Sales.

In equation 2-11, *AE* is the capital using economic activity in employment terms. $TK^{u}(=\sum K_{i}^{u})$ and $TE^{u}(=\sum E_{i}^{u})$ are total capital and total employment in the nation. It is necessary to use *AE* instead of *E* in equation 2-10, because the variation in capital use per employee across industries is very large. The term $FL_{i,t}^{k}$ in equation 2-11 shows relative labor productivity based on labor force availability raised to labor share to reflect labor substitution for capital.

The optimal capital stock for residential housing (*j=2*) is based on the following equation:

$$K_{2,t}^{*k} = \left(\frac{RYD_{t}^{k}}{RYD_{t}^{u}}\right) * K_{2,t}^{*u} * KP_{2}^{k} \qquad (2\text{-}12)$$

Where $\left(\frac{RYD_{t}^{k}}{RYD_{t}^{u}}\right)$ shares out the optimal national residential capital stock, based on the proportion of real disposable income in the region. The optimal capital stock of the nation for type *j (j=1,2)* capital $\left(K_{j,t}^{*u}\right)$ is determined from equation 2-13.

$$K_{j,t}^{*u} = \left(\frac{I_{j,t}^{u}}{\alpha_{j}}\right) + \left(1 - dr_{j,t}^{u}\right) * K_{j,t-1}^{u} \qquad (2\text{-}13)$$

Thus, if we know the speed $\left(\alpha_{j}\right)$ at which investment fills the gaps between the optimal $\left(K_{j,t}^{*u}\right)$ and actual capital stock $\left(K_{j,t}^{u}\right)$, and we know investment in the nation $\left(I_{j,t}^{u}\right)$ and the depreciation rate of capital $\left(dr_{j,t}^{u}\right)$, we can determine the optimal capital stock $\left(K_{j,t}^{*u}\right)$.

Demand for Fuel

Demand for fuel is not explicit in the model. As evident in equation (2-2), the cost of fuel does enter the demands for labor and capital and plays an important role in the model. The treatment of fuel is unique in that the detailed intermediate outputs for oil and gas extraction, coal mining, petroleum and coal products manufacturing, electric power generation, transmission and distribution, and natural gas distribution are excluded from the intermediate industry transactions and treated as a value added factor for purposes of calculating relative costs and labor intensity. As value added factors, fuel, capital, and labor are the Cobb-Douglas substitutes in the production function.

Block 3 – Population and Labor Supply

Population

The population block includes a full cohort-component equation by single year of age, by gender, and by racial/ethnic group. The population at time *t* in region *k* equals the starting population, i.e. the population in the last time period *t-1*, plus components of population change: births, deaths, interregional retired migrants and economic migrants, and international migrants. The components of population change are estimated first based on assumptions of survival rates, fertility rates, and level of net inflow of migrants. When the population estimation is advanced for another year, each age group is updated for one age-year with effects of mortality and interregional and international migration; and a new birth cohort is added in as population of age 0 by applying fertility rates to female population aged 10 to 49. Special population, including military and dependents, prisoners, and college students, do not age. Thus, special population are taken out before aging the population and added back after everyone else is aged.

The population for region *k* at time *t* is

$$N_t^k = N_{t-1}^k + Births_t^k - Deaths_t^k + RTMIG_t^k + ECMIG_t^k + IntMIG_t^k \quad (3\text{-}1)$$

Where;

N_t^k = The population in region *k* at time *t.*

$Births_t^k$ = The number of births during the time period *t-1* to *t* in region *k.*

$Deaths_t^k$ = The number of deaths during the time period *t-1* to *t* in region *k.*

$RTMIG_t^k$ = The net inflow of interregional retired migrants to region *k* during the time period *t-1* to *t*.

$ECMIG_t^k$ = The net inflow of interregional economic migrants to region *k* during the time period *t-1* to *t.*

$IntMIG_t^k$ = The net inflow of international migrants to region *k* during the time period *t-1* to *t.*

Births are determined by applying age-specific fertility rates to the starting female population in each relevant age group, net female international migrants, and net female economic migrants. The international migrants and economic migrants are divided by 2 because they are assumed to have lived in the regional for a half year on average. Births are specific by area and race/ethnicity.

$$Births_t^k = \sum_j^r \sum_c^n \left(N_{c,j,t-1}^{females,k} + \frac{IntMIG_{c,j,t}^{females,k}}{2} + \frac{ECMIG_{c,j,t}^{females,k}}{2} \right) * \left(FR_{c,j,t}^k + BRIPVA_{c,j,t}^k \right) \quad (3\text{-}2)$$

Where;

$N_{c,j,t-1}^{females,k}$ = The female population of age *c (c=10,…,49+)* and race/ethnicity *j (j=1,2,..,4)* at time *t-1* in region *k.*

$IntMIG_{c,j,t}^{females,k}$ = The female international migrants of age *c* and race/ethnicity *j* during the time period *t-1* to *t* in region *k*.

$ECMIG_{c,j,t}^{females,k}$ = The female economic migrants of age *c* and race/ethnicity *j* during the time period *t-1* to *t* in region *k*.

$FR_{c,j,t}^{k}$ =The fertility rate for female population of age *c* and race/ethnicity *j* during the time period *t-1* to *t* in region *k*.

$BRIPVA_{c,j,t}^{k}$ = The policy variable for Birth Rates.

Deaths are determined by applying mortality rates to the sum of starting population, international migrants retired migrants, and economic migrants. Similar to the calculation of births, international migrants, retired migrants, and economic migrants are assumed to have lived in the region for a half year on average. The mortality rate is calculated by 1 minus the survival rate. The estimated deaths are specific by age, racial/ethnic group, and gender.

$$Deaths_t^k = \sum_g^2 \sum_j^r \sum_c^n \left(N_{g,c,j,t-1}^k + \frac{IntMIG_{g,c,j,t}^k}{2} + \frac{RTMIG_{g,c,j,t}^k}{2} + \frac{ECMIG_{g,c,j,t}^k}{2} \right) * \left(1 - \left(SR_{g,c,j,t}^k + SRIPVA_{g,c,j,t}^k\right)\right) \quad (3\text{-}3)$$

Where;

$N_{g,c,j,t-1}^{k}$ = The population of gender *g (g=male, female)*, age *c (c=0,1,…,100+)* and race/ethnicity *j (j=1,2,..,4)* at time *t-1* in region *k*.

$IntMIG_{g,c,j,t}^{k}$ = The international migrants of gender *g*, age *c* and race/ethnicity *j* during the time period *t-1* to *t* in region *k*.

$RTMIG_{g,c,j,t}^{k}$ = The retired migrants of gender *g*, age *c* and race/ethnicity *j* during the time period *t-1* to *t* in region *k*.

$ECMIG_{g,c,j,t}^{k}$ = The economic migrants of gender *g*, age *c* and race/ethnicity *j* during the time period *t-1* to *t* in region *k*.

$SR_{c,j,t}^{k}$ =The survival rate for population of gender *g*, age *c* and race/ethnicity *j* during the time period *t* in region *k*.

$SRIPVA_{g,c,j,t}^{k}$ = The policy variable for Survival Rates.

Retired migrants are based in part by migration patterns for people at and above retirement age 65. In particular a "risk" probability model is used. For areas that experienced an inflow of retired migrants, the probability of a person over age 65 moving into the area is based on the proportion of that population captured in the past. This probability is applied each year in the future to the population age 65 and above in the nation. For areas experiencing net outward migration of the retired population, the past proportion of loss is applied to the number of people in the local area that are age 65 and older. When the data supports it, the above-65 population can be divided into gender and age categories.

In particular, the equation for retired migrants is

$$RTMIG_t^k = \left(rm_t^{IN,k} * (N_t^{65+,u} - N_{t-1}^{65+,k})\right) - \left(rm_t^{OUT,k} * N_{t-1}^{65+.k}\right) + RMIPVA_t^k \quad \text{(3-4)}$$

Where;

$RTMIG_t^k$ = The net inflow or outflow of retired migrants of age *i* (*i*=65,66, …100+) to region *k*

rm_t^k =The net proportion of the relevant population that has historically migrated into or out of area *k*.

$N_{t-1}^{65+,k}$ = The 65 and above population in area *k* for time period *t-1*.

$N_t^{65+,u}$ =The 65 and above population in the nation *u* for time period *t*.

$RMIPVA_t^k$ = The policy variable for Retired Migrants.

The economic migration equation in the model is very important to forecasting the effects of alternative policies. It is based on the assumption that economic migrants will make their migration decisions based on the relative expected after-tax real earned income in alternative locations and the relative amenity attractiveness of these locations.

The migration equation is

$$ECMIG_t^k = \left((\lambda^k + EMPPVA_t^k + \beta_1 \ln(REO_t^k) + \beta_2 \ln(RWR_t^k) + \beta_1 \ln(MIGPROD_t^k)) * LF_{t-1}^k\right) + EMIPVA_t^k \quad \text{(3-5)}$$

Where;

$ECMIG_t^k$ = Net economic migrants (all migrants less than 65 years of age) in area *k*.

LF_{t-1}^k = The labor force for period *t-1* in area *k*.

$$REO_t^k = \left(\frac{\frac{RAE_t^k}{LF_t^k}}{\frac{RAE_t^u}{LF_t^u}}\right) = \text{The relative employment opportunity in area } k \text{ in period } t. \quad \text{(3-6)}$$

RAE_t^k = Residence-adjusted employment in area *k* in period *t*.

If commuter data is available and consistent with the flow of residence adjusted income, residence adjusted employment (*RAE*) is calculated by subtracting gross employees in (GEI) from and adding gross employees out (GEO) to the total number of non-military jobs in the region:

$$RAE_t^k = \left(EMPT_t^k - E_t^{nFM,k}\right) - GEI_t^k + GEO_t^k + NPAPVA_t^k \quad \text{(3-7a)}$$

If no commuter data is available or it is not consistent with the flow of residence adjusted income, residence adjusted employment (*RAE*) is calculated by scaling the non-military jobs in the region by the share of residence adjustment (RA) relative to total labor and proprietor's income (YLPT):

$$RAE_t^k = \left(1 + \left(\frac{RA_t^k}{YLPT_t^k}\right)\right) * \left(EMPT_t^k - E_t^{nFM,k}\right) + NPAPVA_t^k \quad \text{(3-7b)}$$

$MIGPROD_t^k$ = The consumption access index in area *k* in period *t*.

$RWR_t^k = \left(\frac{CR_t^k}{CR_t^u}\right) * \left(\frac{\frac{RYD_t^k}{YP_t^k}}{\frac{RYD_t^u}{YP_t^u}}\right)$ = The relative real compensation rate in area *k* in period *t*. (3-8)

$\overline{CR}_t^k = \sum_{i=1}^{np} \frac{E_{i,t}^k}{TE_t^k} * CR_{i,t}^k$ = Local (*k*) average compensation rate in period *t*. (3-9a)

$\overline{CR}_t^u = \sum_{i=1}^{np} \frac{E_{i,t}^k}{TE_t^k} * CR_{i,t}^u$ = National (*u*) average industry compensation weighted by the employment industry shares in *k* for period *t*. (3-9b)

λ^k =A fixed effect that captures the relative attractiveness of area *k*.

β_1, β_2 = Estimated coefficients.

$EMPPVA_t^k$ = The policy variable for Non-Pecuniary (Amenity) Aspects.

$EMIPVA_t^k$ = The policy variable for Economic Migrants.

The total number of economic migrants is distributed to age, gender, and ethnicity cohorts based on a national distribution.

Labor Force Equations

$$LF_t^k = \sum_{i=l}^{n} PR_{i,t}^k * N_{i,t}^k \quad (3\text{-}10)$$

$$PR_{i,t}^k = \beta_1^k * \left(REA_t^k\right)^{\beta_2} * \left(RWR_t^k\right)^{\beta_3} * PR_{i,t}^u + PRIPVA_t^k \quad (3\text{-}11)$$

Where;

LF_t^k =The labor force in area *k*.

$PR_{i,t}^k$ = The participation rate (i.e. the proportion of the relevant population that is in the labor force) in age cohort *i*, area *k*.

$N_{i,t}^k$ = The number of people in cohort *i*,area *k*.

β_1^k = The fixed effect for area *k*.

β_2, β_3 = The parameters estimated on the basis of pooled or national time series.

$REA_t^k = \left(\frac{EA_t^k}{EA_t^u}\right)$ (3-12)

$EA_t^k = EA_{t-1}^k + \lambda_E\left(EO_t^k - EA_{t-1}^k\right)$

$EA_t^u = EA_{t-1}^u + \lambda_E\left(EO_t^u - EA_{t-1}^u\right)$

EO_t^u = A synthetic labor force based on the local population at fixed national participation rates.

EO_t^k =The residence adjusted employment.

RWR_t^k = The relative real compensation rate.

λ_E = An estimated parameter $0 < \lambda_E < 1$.

The β_2, β_3 values by age cohorts, gender, and racial/ethnic groups have been estimated for 160 (20x2x4) age cohorts in the U.S. The β_1^k parameter is a fixed effect for area k calibrated to the measured labor force (see Treyz, Christopher, and Lou, 1996).

Block 4 – Compensation, Prices and Costs

Production Costs

$$\Omega_{i,t}^{k} = \left[\sum_{j=1}^{np} \left[a_{ji,t}^{k} * CP_{j,t}^{k} \right] + \frac{\left[\left(\frac{CADJ_{i,t}^{k}}{CR_{i,t}^{u}} \right)^{b_{ij,t}^{u}} * \left(RCC_{i,t}^{k} * COSCAP_{i,t}^{k} \right)^{b_{ij,t}^{u}} * \left(\frac{FuelC_{i,t}^{k}}{FuelC_{i,t}^{u}} \right)^{b_{ij,t}^{u}} \right] * \left(1 - \sum_{j=1}^{ns} a_{ij,t}^{u} \right)}{\left(RPRDPV_{i,t}^{k} * \left(RLABPV_{i,t}^{k} \right)^{b_{ij,t}^{u}} \right)} \right] * LAMOMG_{i,T}^{k} * COSPOL_{i,t}^{k} \qquad (4\text{-}1)$$

Where;

$\Omega_{i,t}^{k}$ = The composite cost of production. (This is a composite cost because it incorporates productivity change due to access to material inputs).

$a_{ji,t}^{k} = \left(\frac{a_{ji,t}^{u}}{MCPRODA_{j,t}^{k}} \right)$ = The average *j* purchased per dollar spent on producing *i* in region *k* in time period *t*.

$$CP_{j,t}^{k} = CP_{j,T}^{k} * \left(\frac{CIFP_{j,t}^{k} * MCOST_{j,t}^{k}}{CIFP_{j,T}^{k}} \right) \qquad (4\text{-}2)$$

$CP_{j,t}^{k}$ = The composite price for region *k*, industry *j*, and year *t*.

$CP_{j,T}^{k}$ = The composite input cost based on composite prices calculated in the last history year at the smallest geographic size available.

$CIFP_{j,t}^{k}$ = The delivered average price for region *k*, industry *j*, and year *t*. The local share of the price includes the composite price of production because it is based on the productivity of the inputs due to access to those inputs.

$CIFP_{j,T}^{k}$ = The delivered average price for region *k*, industry *j*, in the last history year.

$$MCOST_{j,t}^{k} = \frac{DD_{j,t}^{k}}{D_{j,t}^{k}} * \left(1 - MPPVM_{j,t}^{k} \right) + MPPVM_{j,t}^{k}$$

$MCOST_{j,t}^{k}$ = A weighted multiplicative policy variable change for Foreign Import Costs.

$DD_{j,t}^{k}$ =The domestic demand for industry *j* in region *k*.

$D_{j,t}^{k}$ =The total demand for industry *j* in region *k*.

$MPPVM_{j,t}^{k}$ = The multiplicative policy variable for Foreign Import Costs.

$CADJ_{i,t}^{k} = \frac{CR_{i,t}^{k} * UECPV_{i,t}^{k}}{\left(\left(\frac{FLA_{i,t}^{k}}{FL_{i,T}^{k}} \right) * FLmult_{i,T}^{k} \right)}$ = The productivity adjusted compensation rate in region *k*.

$CR_{i,t}^{k}$ = The compensation rate of industry *i* in region *k*.

$UECPV_{i,t}^{k}$ = The multiplicative policy variable for Non-Compensation Labor Costs.

$FLA_{i,t}^{k}$ = The moving average of labor productivity in *k* in period *t*.

$FLA_{i,t}^{k} = (1 - \lambda) * FL_{i,t}^{k} + \lambda FLA_{i,t-1}^{k}$

λ = 0.8 = speed of adjustment for moving average.

$FL_{i,T}^{k}$ = Labor productivity due to access by industry *i* in region *k* in the last year of history.

$FLmult_{i,T}^{k}$ = An adjustment to reconcile the aggregated data to the primary source data.

$CR_{i,t}^{u}$ = The compensation rate of industry *i* in the nation.

$b_{ij,t}^{u}$ = Contribution to value added of factor *j*, (labor, capital, and fuel respectively), industry *i*, time *t*.

$RCC_{i,t}^{k}$ = Relative capital cost, industry *i* in region *k*.

$COSCAP_{i,t}^{k}$ = The multiplicative policy variable for Capital Cost.

$FuelC_{i,t}^{k}$ = The weighted cost of fuel of industry *i* in region *k*.

$$FuelC_{i,t}^{k} = \left(\prod_{j=1}^{f} \left(RFuel_{j,jj,t}^{k} * RFCPV_{i,j,t}^{k} \right)^{FVW_{i,j,T}^{S}} \right)$$

$RFuel_{j,jj,t}^{k}$ = The relative cost of fuel by type and category in region *k*.

$RFCPV_{i,j,t}^{k}$ = The policy variable for Fuel Cost by industry *i* and type *j* in region *k*.

$FVW_{i,j,T}^{S}$ = The fuel expenditure weights for industry *i*, type *j*, and state *S* in the last history year.

$FuelC_{i,t}^{u}$ = The weighted cost of fuel of industry *i* in the nation.

$\sum a_{ij,t}^{u}$ = The proportion of all factor inputs in the total inputs into production.

$LAMOMG_{i,T}^{k}$ = An adjustment for aggregation and normalization in the last history year (*T*).

$COSPOL_{i,t}^{k}$ = The multiplicative policy variable for Production Cost.

$RPRDPV_{i,t}^{k}$ = The multiplicative policy variable for Factor Productivity.

$RLABPV_{i,t}^{k}$ = The multiplicative policy variable for Labor Productivity.

Delivered Prices

$$CIFP_{i,t}^{k} = \left[\frac{\prod_{l=1}^{m} \left(\Omega_{i,t}^{l} * \left(ED_{i,t}^{l,k} \right)^{\gamma_i} \right)^{\frac{TIJ_{i,t-1}^{l,k}}{D_{i,t-1}^{k}}}}{\prod_{l=1}^{m} \left(\Omega_{i,t-1}^{l} * \left(ED_{i,t-1}^{l,k} \right)^{\gamma_i} \right)^{\frac{TIJ_{i,t-1}^{l,k}}{D_{i,t-1}^{k}}}} \right] * CIFP_{i,t-1}^{k} \quad (4\text{-}3)$$

Where;

$CIFP_{i,t}^{k}$ = The weighted average of the delivered prices of good *i* sold in region *k* in time period *t*.

$\Omega_{i,t}^{l}$ = The cost of producing output in industry *i* sold in region *l*.

$ED_{i,t}^{l,k}$ = The "effective distance" from *l* to *k* for good *i*.

γ_i = A parameter that is estimated based on observed actual transportation costs.

$TIJ_{i,t-1}^{l,k}$ = The trade flow for good *i* from region *l* to region *k* in the previous time period.

$D_{i,t-1}^{k}$ =The total demand for industry *i* in region *k* in the previous time period.

$CIFP_{i,t-1}^{k}$ = The weighted average of the delivered prices of good *i* sold in region *k* in the previous time period.

Cost of Structures

$$CSTR_{i,t}^{k} = \frac{\left(\frac{(RB^{u}+CSRR^{u})}{(1-UM_{i,t}^{k})}\right)*\left(1-DDFS^{u}-DDMS_{i,t}^{k}-RDMS_{i,t}^{k}+\frac{ZM_{i,t}^{k}}{(RB^{u}+CSRR^{u})}\right)}{\left(\frac{(RB^{u}+CSRR^{u})}{(1-UM^{u})}\right)*\left(1-DDFS^{u}-DDMS^{u}-RDMS^{u}+\frac{ZM^{u}}{(RB^{u}+CSRR^{u})}\right)} \tag{4-4}$$

Where;

$CSTR_{i,t}^{k}$ = Relative structure capital cost for region *k*, industry *i*, and time period *t*.

RB^{u} = National interest rate.

$CSRR^{u}$ = National replacement rate for structures.

$UM_{i,t}^{k}$ = Combined national and regional corporate profit tax rate for region *k*, industry *i*, and time period *t*.

$$UM_{i,t}^{k} = \left(TCP^{u} + \left(TCP^{k} + CAPV_{i,t}^{k}\right)\right) - (TCP^{u} * \left(TCP^{k} + CAPV_{i,t}^{k}\right))$$

TCP^{u} = Federal corporate profit tax rate.

TCP^{k} = Regional corporate profit tax rate.

$CAPV_{i,t}^{k}$ = The additive policy variable for Corporate Profit Tax Rate.

$DDFS^{u}$ = Present value federal depreciation for structures.

$DDMS_{i,t}^{k}$ = Present value depreciation for structures for region *k*, industry *i*, and time period *t*.

$$DDMS_{i,t}^{k} = \left(\frac{\left(TCP^{k} + CAPV_{i,t}^{k}\right) - \left(TCP^{u} * \left(TCP^{k} + CAPV_{i,t}^{k}\right)\right)}{TSLM^{u}}\right) * \left(\frac{\left(1 - e^{-TSLM^{u}*RB^{u}}\right)}{RB^{u}}\right)$$

$TSLM^{u}$ = National structure life time for tax rates.

$RDMS_{i,t}^{k}$ = Present value interest deduction for structures for region *k*, industry *i*, and time period *t*.

$$RDMS_{i,t}^{k} = \left(\frac{UM_{i,t}^{k} * B^{u} * RB^{u}}{(RB^{u} + CSRR^{u})}\right)$$

B^{u} = National proportion of business capital financed by bonds and loans.

$ZM_{i,t}^{k}$ = Effective property tax rate for structures for region *k*, industry *i*, and time period *t*.

$$\begin{aligned} ZM_{i,t}^{k} = TPROP^{k} - \left(TCP^{u} * \left(TPROP^{k} + TPROPP_{t}^{k}\right)\right) \\ - \left(\left(TCP^{k} + CAPV_{i,t}^{k}\right) * \left(TPROP^{k} + TPROPP_{t}^{k}\right)\right) \\ + \left(TCP^{u} * \left(TCP^{k} + CAPV_{i,t}^{k}\right) * \left(TPROP^{k} + TPROPP_{t}^{k}\right)\right) \end{aligned}$$

$TPROP^{k}$ = Regional property tax rate.

$TPROPP_{t}^{k}$ = The additive policy variable for Property Tax Rate.

UM^{u} = Combined national and average state corporate profit tax rate.

$DDMS^{u}$ = Present value depreciation for structures for average state.

$RDMS^{u}$ = Present value interest deduction for structures for average state.

ZM^u = Effective property tax rate for structures for average state.

$$PSTR_t^k = \sum_{i=1}^{np}\left(CP_{i,t}^k * CWSTR_i^u\right) \qquad (4\text{-}5)$$

$PSTR_t^k$ = The cost of purchasing an average unit of structure for region *k* and time period *t*.

$CP_{i,t}^k$ = The composite price for region *k*, industry *i*, and time period *t*.

$CWSTR_i^u$ = The capital weight for structures for industry *i*.

Cost of Equipment

$$CEQP_{i,t}^k = \frac{\left(\frac{(RB^u+CERR^u)}{(1-UM_{i,t}^k)}\right)*\left(1-TIC^u-\left((1-TCP^u)*TIC_t^k\right)-DDFE^u-DDME_{i,t}^k-RDME_{i,t}^k+\frac{WM_t^k}{(RB^u+CERR^u)}\right)}{\left(\frac{(RB^u+CERR^u)}{(1-UM^u)}\right)*\left(1-TIC^u-\left((1-TCP^u)*TICA^u\right)-DDFE^u-DDME^u-RDME^u+\frac{WM^u}{(RB^u+CERR^u)}\right)} \qquad (4\text{-}6)$$

Where;

$CEQP_{i,t}^k$ = Relative equipment capital cost for region *k*, industry *i*, and time period *t*.
RB^u = National interest rate.
$CERR^u$ = National replacement rate for equipment.
$UM_{i,t}^k$ = Combined national and regional corporate profit tax rate for region *k*, industry *i*, and time period *t*.
TIC^u = National investment tax credit.
TCP^u = Federal corporate profit tax rate.
TIC_t^k = Investment tax credit for region *k* and time period *t*.
$DDFE_t^u$ = Present value federal depreciation for equipment for time period *t*.
$DDME_{i,t}^k$ = Present value depreciation for equipment for region *k*, industry *i*, and time period *t*.

$$DDME_{i,t}^k = \left(\frac{\left(TCP^k + CAPV_{i,t}^k\right) - \left(TCP^u * \left(TCP^k + CAPV_{i,t}^k\right)\right)}{TELM^u}\right) * \left(\frac{\left(1 - e^{-TELM^u * RB^u}\right)}{RB^u}\right)$$

TCP^k = Regional corporate profit tax rate.
$CAPV_{i,t}^k$ = The additive policy variable for Corporate Profit Tax Rate.
$TELM^u$ = National equipment life time for tax rates.

$RDME_{i,t}^k$ = Present value interest deduction for equipment for region *k*, industry *i*, and time period *t*.

$$RDME_{i,t}^k = \left(\frac{UM_{i,t}^k * B^u * RB^u}{(RB^u + CERR^u)}\right)$$

B^u = National proportion of business capital financed by bonds and loans.

WM_t^k = Equipment tax for region *k* and time period *t*.

$$WM_t^k = TEQP^k - (TCP^u * TEQP^k)$$

$TEQP^k$ = Regional equipment tax rate.

UM^u = Combined national and average state corporate profit tax rate.
$TICA^u$ = Average state investment tax credit.
$DDME^u$ = Present value depreciation for equipment for average state.
$RDME^u$ = Present value interest deduction for equipment for average state.
WM^u = Equipment profit tax for average state.

$$PEQP_t^k = \sum_{i=1}^{np}\left(CP_{i,t}^k * CWEQP_i^u\right) \quad (4\text{-}7)$$

$PEQP_t^k$ = The cost of purchasing an average unit of equipment for region *k* and time period *t*.
$CP_{i,t}^k$ = The composite price for region *k*, industry *i*, and time period *t*.
$CWEQP_i^u$ = The capital weight for equipment for industry *i*.

Cost of Inventory

$$CINV_{i,t}^k = \frac{\left(\frac{(RB^u)}{(1-UM_{i,t}^k)}\right)*\left(1-\frac{(UM_{i,t}^k*B^u*RB^u)}{RB^u}\right)}{\left(\frac{(RB^u)}{(1-UM^u)}\right)*\left(1-\frac{(UM^u*B^u*RB^u)}{RB^u}\right)} \quad (4\text{-}8)$$

Where;

$CINV_{i,t}^k$ = Relative inventory capital cost for region *k*, industry *i*, and time period *t*.
RB^u = National interest rate.
$UM_{i,t}^k$ = Combined national and regional corporate profit tax rate for region *k*, industry *i*, and time period *t*.
UM^u = Combined national and average state corporate profit tax rate.
B^u = National proportion of business capital financed by bonds and loans.

Cost of Capital

$$RCC_{i,t}^k = \left(\left(CSTR_{i,t}^k * PSTR_t^k\right)^{CS_i^u} * \left(CEQP_{i,t}^k * PEQP_t^k\right)^{CE_i^u} * \left(CINV_{i,t}^k\right)^{CI_i^u} * \left(PH_t^k * NMPVH_t^k\right)^{CL_i^u}\right) * COSCAP_{i,t}^k \quad (4\text{-}9)$$

Where;

$RCC_{i,t}^k$ = Relative capital cost for region *k*, industry *i*, and time period *t*.
$CSTR_{i,t}^k$ = Relative structure capital cost for region *k*, industry *i*, and time period *t*.
$PSTR_t^k$ = The cost of purchasing an average unit of structure for region *k* and time period *t*.
CS_i^u = National proportion of capital accounted for by structures for industry *i*.
$CEQP_{i,t}^k$ = Relative equipment capital cost for region *k*, industry *i*, and time period *t*.
$PEQP_t^k$ = The cost of purchasing an average unit of equipment for region *k* and time period *t*.
CE_i^u = National proportion of capital accounted for by equipment for industry *i*.
$CINV_{i,t}^k$ = Relative inventory capital cost for region *k*, industry *i*, and time period *t*.
CI_i^u = National proportion of capital accounted for by inventory for industry *i*.
PH_t^k = Relative housing price in region *k* for time period *t*.
$NMPVH_t^k$ = The multiplicative policy variable for Housing and Land Price.
CL_i^u = National proportion of capital accounted for by land for industry *i*.
$COSCAP_{i,t}^k$ = The multiplicative policy variable for Capital Cost.

Consumption Deflator

For consumption category *j* in time *t* we assume Cobb-Douglas substitutability of the sectors that are inputs into this consumption commodity.

$$CIFP_{j,t}^{k} = CIFP_{j,t}^{u} * \prod_i \left(CIFP_{i,t}^{k}\right)^{PCE_{ij,t}^{u}} * CPPV_{j,t}^{k} \quad (4\text{-}10)$$

Where;

$CIFP_{j,t}^{k}$ = The delivered consumer price of consumption commodity *j* in time *t* in region *k*.
$CIFP_{j,t}^{u}$ = The average delivered consumer price of consumption commodity *j* in time *t* in the nation.
$CIFP_{i,t}^{k}$ = The delivered price of industry *i* in time *t* in region *k*.
$PCE_{ij,t}^{u}$ = The proportion of commodity *j* obtained from industry *i*.
$CPPV_{j,t}^{k}$ = The multiplicative policy variable for Consumer Price.

Consumer Price Index Based on Delivered Costs

$$CPI_t^k = \left(\prod_{j=1}^{ncomm} \left(\frac{CIFP_{j,t}^{k}}{CIFP_{j,t-1}^{k}}\right)^{WC_{j,t-1}^{u}}\right) * CPI_{t-1}^{k} \quad (4\text{-}11)$$

Where;

CPI_t^k = The consumer price index in region *k* and time period *t*.
$CIFP_{j,t}^{k}$ = The consumer price of commodity *j* in region *k* and time period *t*.
$CIFP_{j,t-1}^{k}$ = The consumer price of commodity *j* in region *k* and the previous time period.
$WC_{j,t-1}^{u}$ = The proportion of commodity *j* in total national consumption for the previous time period.
CPI_{t-1}^{k} = The consumer price index in region *k* and the previous time period.

Consumer Price to be Used for Potential In or Out Migrants

$CPIPH_t^k$ = Equation (4-11) with the housing cost replaced by relative price of purchasing a house.
$CIFP_{j,t}^{k} = PH_t^k$

Where;

$CPIPH_t^k$ =The cost of living in area *k* when the relative price of buying a new house is used in the consumer price index for housing costs.
PH_t^k = Relative housing price at time *t* in area *k*.

Housing Price Equation

The REMI housing price equation has two coefficients for all regions in the model: the estimated elasticity of response to a change in real disposable income and the estimated elasticity of response to a change in population. Both of these coefficients are currently based on state or metropolitan-level averages and used as standard default elasticity measurements evident in the Housing Price equation below.

$$PH_t^k = \left(\left(\varepsilon_1 \left(\frac{\frac{RYD_t^k}{RYD_t^u}}{\frac{RYD_{t-1}^k}{RYD_{t-1}^u}} - 1 \right) + \varepsilon_2 \left(\frac{\frac{N_t^k}{N_t^u}}{\frac{N_{t-1}^k}{N_{t-1}^u}} - 1 \right) \right) + 1 \right) * PH_{t-1}^k * NMPVH_t^k \qquad (4\text{-}12)$$

PH_t^k = Relative housing price in region *k* for time period *t*.

ε_1 = The estimated (or user-entered) elasticity of response to a change in real disposable income.

RYD_t^k = Real disposable income in region *k* for time period *t*.

RYD_t^u = Real disposable income in the nation for time period *t*.

RYD_{t-1}^k = Real disposable income in region *k* for the previous time period.

RYD_{t-1}^u = Real disposable income in the nation for the previous time period.

ε_2 = The estimated (or user-entered) elasticity of response to a change in population.

N_t^k = Population in region *k* for time period *t*.

N_t^u = Population in the nation for time period *t*.

N_{t-1}^k = Population in region *k* for the previous time period.

N_{t-1}^u = Population in the nation for the previous time period.

PH_{t-1}^k = Relative housing price in region *k* for the previous time period.

$NMPVH_t^k$ = The multiplicative policy variable for Housing and Land Price.

The values of ε_1 and ε_2 are estimated for each state and metropolitan area through a regression analysis that compares the housing price changes to the number of houses using data from a historical time series. The user may also enter alternative values.

The region-specific approach estimates price responses to changes in demand, which vary by state or metropolitan-level area. Changes in demand have been estimated using building permit and housing unit data from Freddie Mac, Conventional Mortgage Home Price Index, State Indices.

The region-specific approach scales the previously estimated national housing price response according to the proportion of the regions' price response to the average national price response. This may more accurately reflect the regions' change in demand, and will therefore yield a more accurate forecast.

The Compensation Equation

The final form of the compensation rate (*CR*) equation for area *k* is

$$CR_{i,t}^{k} = \left((1 + \Delta CRD_{i,t}^{k})(1 + k_t^u)\right) * CR_{i,t-1}^{k} * MWAPV_{i,t}^{k} \quad (4\text{-}13)$$

Where;

$CR_{i,t}^{k}$ = Compensation rate in industry *i* for region *k* in time period *t*.

$\Delta CRD_{i,t}^{k}$ = The predicted change in the compensation rate in industry *i* due to changes in demand and supply conditions in the labor market in area *k*.

k_t^u = The change in the national compensation rate that cannot be explained by changes in the national average compensation rate for all industries, which is due to change in demand and supply conditions and to industry mix changes in the nation.

$CR_{i,t-1}^{k}$ = Compensation rate in industry *i* for region *k* in the previous time period.

$MWAPV_{i,t}^{k}$ = The multiplicative policy variable for Compensation Rate.

$$\Delta CRD_{i,t}^{k} = \alpha_1 \left[\left(\frac{\frac{E_t^k}{LF_t^k}}{\frac{EA_t^k}{LFA_t^k}}\right) - 1\right] + \alpha_2 \left[\left(\frac{EO_{i,t}^{k}}{EOA_{i,t}^{k}}\right) - 1\right] \quad (4\text{-}14)$$

α_1 = Estimated parameter using pooled time series data.

$E_t^k = \sum_{i=1}^{ns} E_{i,t}^{k}$ = Total employment in region *k* for time period *t*.

LF_t^k = The labor force in region *k* for time period *t*.

EA_t^k = The moving average of total employment in region *k* for time period *t*.

$EA_t^k = (1-\lambda)E_t^k + \lambda EA_{t-1}^{k}$

LFA_t^k = A geometrically declining moving average of the labor force in region *k* for time period *t*.

$LFA_t^k = (1-\lambda)LF_t^k + \lambda LFA_{t-1}^{k}$

λ = 0.8 = speed of adjustment for moving average

α_2 = Estimated parameter using pooled time series data.

$\left(\frac{EO_{i,t}^{k}}{EOA_{i,t}^{k}}\right) = \sum_{j=1}^{q} d_{j,i}^{u} \left(\frac{EO_{j,t}^{k} - OTRPV_{j,t}^{k}}{EOA_{j,t}^{k}}\right)$

$\left(\frac{EO_{i,t}^{k}}{EOA_{i,t}^{k}}\right)$ = The demand relative to past demand for the occupations used by industry *i*.

$EOA_{j,t}^{k} = (1-\lambda)EO_{j,t}^{k} + \lambda EOA_{j,t-1}^{k}$

$d_{j,i}^{u}$ = Occupation *j*'s proportion of industry *i*.

$OTRPV_{j,t}^{k}$ = The policy variable for Occupational Training.

$$\Delta CRD_{i,t}^{u} = \alpha_1 \left[\left(\frac{\frac{E_t^u}{LF_t^u}}{\frac{EA_t^u}{LFA_t^u}} \right) - 1 \right] + \alpha_2 \left[\left(\frac{EO_{i,t}^u}{EOA_{i,t}^u} \right) - 1 \right] \quad (4\text{-}15)$$

Then, it is possible to predict the demand and supply effect on national compensation and thus determine the national compensation change by industry.

Since

$$CR_{i,t}^{u} = \left(1 + \Delta CRD_{i,t}^{u}\right) * CR_{i,t-1}^{u} \quad (4\text{-}16)$$

The average compensation in year *t* in the nation, taking into account the change in the mix of industries as well as demand and supply labor market conditions, can be calculated as follows:

$$CRDM_t^u = \sum_{j=1}^{ns} \left(\frac{E_{i,t}^u}{E_t^u} \right) \left(1 + \Delta CRD_{i,t}^{u}\right) * CR_{i,t-1}^{u} \quad (4\text{-}17)$$

Where;

$CRDM_t^u =$ The average compensation in the year *t* based on year *t* compensation mix changes, demand change for occupations, and demand vs. supply in the labor market.

$E_{i,t}^u =$ Employment in industry *i* in period *t* in the nation.

$E_t^u = \sum_{i=1}^{ns} E_{i,t}^u =$ Total employment in the nation for time period *t*.

Then k_t^u is determined as:

$$k_t^u = \left(\frac{\left(\frac{COMP_t^u}{E_t^u}\right) - CRDM_t^u}{\left(\frac{COMP_{t-1}^u}{E_{-1}^u}\right)} \right) * \left(\frac{\left(\frac{\sum E_{i,t}^u * CR_{i,t-1}^u}{E_{t-1}^u}\right)}{\left(\frac{\sum E_{i,t-1}^u * CR_{i,t-1}^u}{E_t^u}\right)} \right) \quad (4\text{-}18)$$

Where;

$COMP_t^u =$ Total compensation in the nation in time period *t*.

$k_t^u =$ All national compensation changes not represented by changes in industry mix and labor market demand and supply conditions, relative to the hypothetical average compensation in *t-1*, using the national compensation rate for each industry in year *t-1* and the current year's industry mix. This value, *k*, is then used in equation (4-13) to align the weighted average of the compensation changes over all of the component regions within the nation. Thus, the local areas will then reflect determinants of compensation changes, such as changes in labor market legislation, increased union militancy, cost of living adjustments, etc., at the national, which are not due to labor force supply and demand changes or industry shifts.

The Wage and Salary Disbursements Equation

The wage equation follows the same form as the compensation equation, but the $\alpha_1$and α_2 parameters have been estimated separately so have different values.

The Earnings by Place of Work Equation

The earnings equation follows the same form as the compensation equation, but the $\alpha_1$and α_2 parameters have been estimated separately so have different values.

Block 5 - Market Shares

$$s_{i,t}^{k,l} = \frac{DQ_{i,T}^{k} * SALPOLM_{i,t}^{k} \left(\frac{\Omega A_{i,t}^{k}}{\Omega A_{i,T}^{k}}\right)^{1-\sigma_i} \left(IMIX_{i,t}^{k}\right)^{\lambda_i} \left(ED_i^{k,l}\right)^{-\beta_i}}{\sum_{j=1}^{m} DQ_{i,T}^{j} \left(\frac{\Omega A_{i,t}^{j}}{\Omega A_{i,T}^{j}}\right)^{1-\sigma_i} \left(IMIX_{i,t}^{j}\right)^{\lambda_i} \left(ED_i^{j,l}\right)^{-\beta_i}} \tag{5-1}$$

$s_{i,t}^{k,l}$ = The share of the domestic demand in area *l* supplied by area *k*, for industry *i* in time period *t*.

$DQ_{i,T}^{k}$ = Domestic output in the last history year.

T = As a subscript, indicates the last history year.

$\Omega A_{i,T}^{k}$ = The cost of production in *k* in the last history year.

$\Omega A_{i,t}^{k}$ = The moving average of the cost of production in *k*.

$$\Omega A_{i,t}^{k} = (1-\lambda)\Omega_{i,t}^{k} + \lambda \Omega A_{i,t-1}^{k} \tag{5-2}$$

λ = 0.8 = speed of adjustment for moving average

$ED_i^{k,l}$ = An effective distance equivalent to calibrate the model to detailed balanced trade flows at a low geographic level.

β_i = The distance decay parameter in a gravity model.

σ_i = The estimated price elasticity.

$SALPOLM_{i,t}^{k}$ = The multiplicative policy variable for Firm Sales.

λ_i = A parameter between $0 < \lambda_i < 1$, as estimated econometrically, that shows the effect of the detailed industry mix on the change in *k*'s share of the market due to differential growth rates predicted in the nation for the detailed industry and the difference in *k*'s participation in these industries relative to the nation (see *IMIX* below).

For *l*=1,…*m* and *n* is the number of sub-national regions in the model. The value for σ_i is calculated by isolating movements along the demand curve. The movement along the curve yields an elasticity of substitution (σ_i) estimate. These estimates are obtained from a pooled non-linear search over all regions. The β_i value is found using a dynamic search for the distance decay parameter in a gravity model for each industry.

$$IMIX_{I,t}^{k} = \left\{ \frac{\prod_{i \in I} \left(\frac{Q_{i,t}^{u}}{Q_{i,t-1}^{u}}\right)^{WI_{i,t-1}^{k}}}{\prod_{i \in I} \left(\frac{Q_{i,t}^{u}}{Q_{i,t-1}^{u}}\right)^{WI_{i,t-1}^{u}}} \right\} * IMIX_{I,t-1}^{k} \tag{5-3}$$

$$wi_{i,t-1}^{k} = \left(\frac{Q_{i,t-1}^{k}}{\sum_{i \in I} Q_{i,t-1}^{k}}\right) \qquad wi_{i,t-1}^{u} = \left(\frac{Q_{i,t-1}^{u}}{\sum_{i \in I} Q_{i,t-1}^{u}}\right)$$

$IMIX_{I,T}^{k} = 1$

$IMIX_{I,t}^{k}$ = A variable using local shares at a detailed level in the numerator applied to national growth rates, and shares in the denominator applied to the same rates. Equals 1 if no detailed industry or forecasts are available.

$$sx_{i,t}^{k,row} = \frac{X_{i,T}^{k,row}}{X_{i,T}^{u,row}} * \left(\frac{\Omega A_{i,t}^{k}}{\Omega A_{i,T}^{k}} * XPPVM_{i,t}^{k}\right)^{1-\sigma_i} \quad (5\text{-}4)$$

Where;

$sx_{i,t}^{k,row}$ = Area k's share of national exports to the rest of the world (*row*).

$X_{i,T}^{k,row}$ = Area k's exports to the rest of the world in the last history year (T).

$X_{i,T}^{u,row}$ = The nation's exports to the rest of the world in the last history year (T).

$\Omega A_{i,t}^{k}$ = A moving average (with geometrically declining weights) of the relative cost of production in time period t (T if the last history year of the series).

σ_i = The estimated price elasticity.

$XPPVM_{i,t}^{k}$ = The multiplicative policy variable for Foreign Export Costs.

$$sd_{i,t}^{k} = 1 - \left(\left(\frac{\frac{M_{i,T}^{k,row}}{M_{i,T}^{u,row}} * M_{i,t}^{u,row}}{D_{i,T}^{k}}\right) * \left(\frac{\Omega A_{i,T}^{k}}{\Omega A_{i,t}^{k}}\right)^{1-\sigma_i} * \left(\frac{\frac{D_{i,t}^{k}}{D_{i,t}^{u}}}{\frac{D_{i,T}^{k}}{D_{i,T}^{u}}}\right)\right) \quad (5\text{-}5)$$

Where;

$sd_{i,t}^{k}$ = The share of area k's demand for good i that is supplied from within the nation.

$M_{i,T}^{k,row}$ = Area k's imports from the rest of the world in the last history year (T).

$M_{i,T}^{u,row}$ = Imports of i into the nation (u) in the last history year (T).

$M_{i,t}^{u,row}$ = Imports of i into the nation (u) in time period t.

$\Omega A_{i,t}^{k}$ = A moving average (with geometrically declining weights) of the relative cost of production in time period t (T if the last history year of the series).

$D_{i,t}^{k}$ =The total demand for industry i in region k and time period t.

$D_{i,T}^{k}$ =The total demand for industry i in region k in the last history year (T).

$D_{i,t}^{u}$ =The total demand for industry i in the nation in time period t.

$D_{i,T}^{u}$ =The total demand for industry i in the nation in the last history year (T).

σ_i = The estimated price elasticity.

For further information about the incorporation of the new economic geography as shown in this section and in section 4 above, please see Fan, Treyz, and Treyz, 2000.

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